

Contingent Payment Mechanisms to Maximize Resource Utilization *

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Abstract

We study the fundamental problem of assigning resources to maximize their expected utilization. This kind of problem arises in many settings, where it is in the interest of a group, the general public, or a principal, that assigned resources be used and not wasted. At the time of assigning resources, each agent privately holds information about her distribution on future values for different assignments. Subsequently, we allow for payments to be contingent on *ex post* utilization decisions. The objective is to maximize expected utilization, while insisting on dominant-strategy equilibria, individual rationality, and no-deficit. We also consider two additional, natural properties: an agent should not pay if she utilizes a resource, and resources should always be assigned if possible. We show that the natural, *contingent second-price* (CSP) mechanism, is unique under these criteria and optimal for a larger class of mechanisms for assigning a single resource. We extend the CSP mechanism to assign multiple, heterogeneous resources (making a technical connection with assignment problems under non quasi-linear utility functions), showing that this generalized mechanism outperforms other methods such as first-come-first-serve and the VCG mechanism.

1 Introduction

Allocated resources often go to waste, even when in scarce supply. It is common in university departments, for example, to find that all rooms are fully booked in advance, yet walking down the corridor one sees that many rooms are in fact empty. For examples from different domains, consider allocating spots in a spinning class to gym members, and assigning time slots for a public electric vehicle charging station to residents in a neighborhood. Even a gym member who is highly uncertain about his ability to attend the class, or a resident unsure about her need for charging, may selfishly reserve a space just in case this turns out to be convenient.

Resource Allocation under Uncertainty What is common in these problems is the presence of uncertainty and a subsequent, *ex post* decision about utilization by allocated agents. At the time when assignments need to be made (period zero), each agent has uncertainty in her value for using an assigned resource or even whether this will be possible at all. The uncertainty is not resolved

*The authors thank Rakesh Vohra, Ilya Segal, Scott Kominers, Di Pei, Thibaut Horel and Debmalya Mandal for helpful discussions.

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until some time in the future (period one), at which point we assume it is too late to assign an unused resource to another agent. Our focus in this paper is on two period problems, but more general scenarios need not be limited to two periods: gym classes may take place every week, and people’s uncertainty about attending may reduce gradually between the reservation and the actual time of each class.

The common practice of giving resources to agents for free (randomly, or on a first-come-first-serve basis) wastes resources, even though high utilization may be the first-order objective: universities may define what they consider to be high value uses, and beyond that want their facilities to be utilized rather than reserved and wasted. Similarly, a gym manager may have a preference for higher attendance in classes to retain and attract more members to the gym; and the ongoing funding for public charging stations may rely on them being used. In some settings, under-utilization can also have broader, negative effects on society. For example, less electric vehicle charging means more pollution, or to consider another domain, unused and underdeveloped wireless spectrum means a lost opportunity for broad spillover effects to the rest of the economy.¹

Despite appearing important to practice and simple to state, the problem of optimal mechanism design for utilization does not appear to have been formally defined in the literature. The naïve solution of collecting bids and running the second price (SP) auction (and more generally, the VCG mechanism) does not necessarily assign resources to the more reliable agents, since higher bids need not reflect higher utilization. Moreover, the payments are collected in period zero and thus “sunk,” and do not shape incentives downstream towards higher utilization. Another worry is that in many scenarios of interest there appears to be a “no-charge” norm so that agents should not be charged if they act as intended; e.g., a gym member should not be charged for attending classes when she has already paid for the gym membership, and residents should not be charged for using the community’s electric vehicle charging stations.

Payments that are contingent on utilization decisions have been used in practice to disincentivize no-shows: hospitals charge patients penalties that are not covered by insurance for missing appointments; organizers of conferences reimburse students their registration fee if they attend talks; and fancy restaurants charge a fee if guests who reserved don’t show up. But we are not aware of any formal analysis of such *ad hoc* mechanisms, or a design approach that takes into account the penalty that an individual participant would be willing to face. This is the main conceptual contribution of our paper. Rather, existing approaches can be viewed as simple, first-come first-served schemes, and where it is not clear how the penalty should be set: a penalty that is too small is not effective, whereas a penalty that is too big will drive away participation in the scheme. Perfect utilization could of course be achieved through always charging the assigned agent a very large penalty for no-show, or paying a very large bonus for using a resource, but this would drive away participants or run into a deficit.

Contingent Payment Mechanisms for Utilization Maximization In this paper, we formalize the utilization-optimal mechanism design problem, and define a class of *contingent payment mechanisms* that make use of payments that are contingent on agents’ publicly observable *ex post* utilization decisions. In period zero, agents make reports based on their distributional information about their future values for resources (the agent types). The mechanism assigns the resource or

¹In regard to welfare considerations— a standard second-price auction (SP) would maximize agents’ expected welfare for single resource allocation. If, in addition, we model society as gaining $C > 0$ from a resource being utilized, then modifying SP by adding a penalty C if the resource is not used, and allowing negative bids (i.e. allowing positive transfers from the mechanism to the agents) is welfare-optimal. But this is not a satisfactory solution to our problem because it runs a deficit. In fact, with large C and insisting on no-deficit, we can prove that the welfare optimal mechanisms are precisely those that are utilization maximizing (see Appendix D).

resources and determine the contingent payments for each assigned agent. When period one comes, each agent privately observes her realized value and decides whether to use an assigned resource based on this value as well as the contingent payments. The design objective is to maximize utilization (i.e. the expected number of resources utilized) in a dominant strategy equilibrium. We also insist on voluntary participation by agents and that the mechanism is no-deficit, thus precluding always charging very large penalties and paying very large rewards.

Given that types correspond to value distributions, this is a problem of multi-dimensional mechanism design. As would be expected from the literature, this presents technical challenges. One way to see the impact of multi-dimensional types is to consider that the payments an assigned agent faces can have two parts, consisting of one payment when she uses the resource and one when she does not. Because of the rich types, different types have different probabilities for using the resource, and thus different probabilities of needing to pay each part of the payments, and as a consequence, different preference orderings over these payments. The present problem is challenging, also, because we must address information asymmetries both before and after the time of contracting, value distributions and realized values, respectively.

Our Contributions For single resource allocation, we study the natural *contingent second price (CSP) mechanism*, which elicits a single bid from each agent, allocates to the highest bidder, and charges her the second highest bid only if she does not use the resource. As an indirect mechanism, CSP avoids the inconvenience of eliciting full value distributions. We show that it is a dominant strategy in CSP to bid an amount that is equal to a the maximum no-show penalty an agent is willing to pay (Theorem 1), and that the utilization achieved by CSP is always higher than that under the SP auction (Theorem 2.) This is because CSP selects more reliable agents than SP, since a high bid indicates an agent is more reliable and less likely to pay a penalty. Moreover, the use of contingent payments further promotes allocated agents to use the resources— shaping behavior of an agent in period one and promoting an agent to use the resource with higher probability.

In addition to the “no-charge” property, (agents that act as intended do not make a payment), we adopt the natural “always allocate” property (i.e., the resource is always allocated to some agent.) Given this, we prove that the CSP mechanism is the only dominant strategy, individually rational, anonymous, and deterministic mechanism that does not run into deficit, always allocates and satisfies no-charge (Theorem 3.) Moreover, CSP is not dominated by any mechanism (Theorem 5) if we relax the last two conditions, and is provably optimal in this class of mechanisms for the simple model of agents types where each agent gets a fixed value from using the resource, but is only able to do so with a fixed probability smaller than one. We also provide both theoretical results and detailed simulation results to establish that for simple type distributions, relaxing the last two conditions does not improve utilization on average.

In Section 5, we generalize the CSP mechanism to assign multiple resources. For allocating m identical resources, we prove that the contingent $(m+1)^{\text{st}}$ price mechanism is the only mechanism with the set of desired criteria, and is again optimal among a larger set of mechanisms (Theorem 6.) Since agents’ expected utilities are not quasi-linear in contingent payments, the generalization of CSP to allocate multiple, heterogeneous resources makes use of the minimum competitive equilibrium price mechanism for non-quasi-linear assignment [12, 1]. We present simulation results in Section 6 and in the appendix, comparing utilization achieved by this generalized CSP mechanisms with that of the VCG mechanism and other benchmarks. We show that a significant improvement in utilization is achieved by the generalized-CSP mechanism, and that allowing reserve prices does not improve utilization on average.

1.1 Related Work

Contingent payments have arisen in previous work on auction design. Prominent examples include auctioning oil drilling licenses [18], royalties [7, 10], ad auctions [26], and selling a firm [13]. In such auctions, payments are contingent on some observable world state (e.g. amount of oil produced, a click, or the ex post cash flow) rather than on an agent’s own downstream actions. Thus, this is a departure from our model. Moreover, the major role of contingent payments in these applications is to improve revenue and hedge risk [24], rather than providing bidders with a way to signal their own, idiosyncratic uncertainty and thus address moral hazard.

There’s an extensive literature on strictly-proper scoring-rules [15], but this does not appear to be helpful for eliciting the information about uncertainty because (i) only the actions, and not realized values are observed, and thus a scoring-rule method could not be used to elicit beliefs about value distributions, and (ii) the utility for using an assigned resource is entangled with the incentives to provide accurate prediction about one’s utilization action.

Our problem is a kind of principal-agent problem [17, 19]. Classically, this literature addresses problems with hidden information (e.g. seller’s quality [11]) before the time of contracting— this is the problem of *adverse selection*. In addition, this literature addresses problems for which information asymmetry arises after the time of contracting (e.g. shipping a low quality good)— this is the problem of *moral hazard*. The distinction between the two settings is blurred in dynamic settings (see [25, 4],) as is the case for our problem where the asymmetries exist both before and after contracting. Although agents’ actions are fully observable, uncertainty together with participation constraints make it impossible to charge unbounded penalties, which is a standard approach when actions are observable in settings with moral hazard. We are aware of no model or methods in the principal-agency literature that addresses our problem.

On mechanisms with actions after the time of contracting, Atakan and Ekmecki [2] study auctions where the value of taking each action depends on the collective actions by all allocated agents, but these actions are taken before rather than after observing the world state and thus the timing of information is quite different than our model. Courty and Li [9] study the problem of revenue maximization in selling airline tickets, where passengers have uncertainty about their value for a trip at the time of booking, and decide whether to take a trip only after realizing their actual values. Although Courty and Li [9] model agents’ types as distributions, and the optimal mechanism in their setting can be understood as a menu of contingent contracts, the type spaces in their model are effectively one-dimensional (because they require stochastic dominance conditions).

The closest related work is on the design of mechanisms for incentivizing reliability in the specific setting of demand-side response in electric power systems [20, 21], where selected agents decide whether to respond only after uncertainty in their costs for demand response are resolved. The objective there is to guarantee a probabilistic target on the collective actions taken by agents, without selecting too large a number of agents or incurring too much of a total cost. In contrast to the models in the present paper, there is no hard feasibility constraint in the setting of demand response— that is, whereas only one agent can be assigned to each one of our resources, in demand response any number between zero and all agents can reduce demand. This makes the present problem more challenging.

Other papers study dynamic mechanism design in assignment settings, including models with the possibility that workers assigned to tasks will prove to be unreliable [23] and problems where the goal is to maximize expected, discounted value in the presence of uncertainty [22, 6, 3]. But we are not aware of any work in the dynamic mechanism design literature that is applicable to our problem, which can be construed as the limit case in which the principal has a very large value (representing society) for agents choosing to utilize the assigned resources.

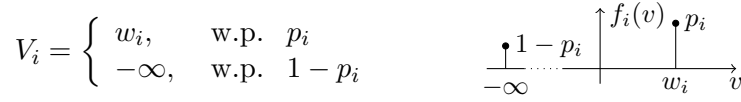


Figure 1: Agent value distribution in the (w_i, p_i) type model.

2 Preliminaries

We first introduce the model for the assignment of a single resource. There is a set $N = \{1, 2, \dots, n\}$ of agents and two time periods. In period 0, the value of agent i for using the resources is uncertain, represented by a random variable V_i , whose exact (potentially negative) value is not realized until period 1. The cumulative distribution function (CDF) F_i of V_i is agent i 's private information at period 0, and corresponds to her *type*. Let $F = (F_1, \dots, F_n)$ denote a type profile.

The assignment is determined in period 0, whereas the allocated agent decides on whether to use the resource at period 1, after she privately learns the realization v_i of V_i . Define $V_i^+ \triangleq \max\{V_i, 0\}$. We make the following assumptions about F_i :

- (A1) $\mathbb{E}[V_i^+] > 0$, which means V_i takes positive value with non-zero probability. An agent for which this is violated would never be interested in the resource.
- (A2) $\mathbb{E}[V_i^+] < +\infty$, which means agents do not get infinite utility from the resource.
- (A3) $\mathbb{E}[V_i] < 0$, which means the hard commitment of “always use the resource no matter what happens” is not favorable.

(A1) and (A2) are clearly reasonable. (A3) may seem like a strong requirement, however, we will see later that an agent for whom (A3) is violated would be willing to commit to pay an unbounded penalty if the resource is not used, which we don't consider to be reasonable. The following is an example value distribution with discrete support and models an agent who may be completely unable to use the resource. See also Example 5 in Section 4 for an example type model where values are continuously distributed.

Example 1 ((w_i, p_i) model). The value for agent i to use the resource is $w_i > 0$, however, she is able to do so only with probability $p_i \in (0, 1)$. With probability $1 - p_i$, agent i is unable to show up to use the resource due to a hard constraint. The hard constraint can be modeled as V_i taking value $-\infty$ with probability $1 - p_i$. See Figure 1. We have $\mathbb{E}[V_i^+] = w_i p_i > 0$ and $\mathbb{E}[V_i] = -\infty < 0$, thus (A1)-(A3) are satisfied. The $-\infty$ value is not critical for (A3) — it suffices if with probability $1 - p_i$, the agent incurs a large enough cost if she forces herself to show up.

2.1 Contingent Payment Mechanisms

At period 0, each agent makes a report r_i from some set of messages \mathcal{R} . Let $r = (r_1, \dots, r_n) \in \mathcal{R}^n$ denote a report profile. A mechanism is defined by $\mathcal{M} = (\mathcal{R}, x, t, t^{(0)}, t^{(1)})$. Based on the reports, an *allocation rule* $x = (x_1(r), \dots, x_n(r)) : \mathcal{R}^n \mapsto \{0, 1\}^n$ allocates the right to use the resource to at most one agent, which we denote as $i^* = i^*(r)$ s.t. $x_{i^*}(r) = 1$. Each agent is charged $t_i(r)$ in period 0. The allocated agent i^* is also charged $t_{i^*}^{(0)}(r)$ or $t_{i^*}^{(1)}(r)$ at the end of period 1, depending on her action of not using or using the resource, respectively.

The timeline of a contingent payment mechanism is as follows:

Period 0:

- Each agent i reports r_i to the mechanism based on knowledge of type F_i .
- The mechanism allocates the resource to agent i^* .
- The mechanism collects $t_i(r)$ from each agent, and determines the contingent payments $t_{i^*}^{(0)}(r)$, $t_{i^*}^{(1)}(r)$ for the allocated agent.

Period 1:

- The allocated agent privately observes the realized value v_{i^*} of V_{i^*} .
- The allocated agent decides which action to take based on v_{i^*} and $t_{i^*}^{(0)}(r)$, $t_{i^*}^{(1)}(r)$.
- The mechanism collects a contingent payment from i^* based on her action.

We assume that agents are risk-neutral, expected-utility maximizers with quasi-linear utility functions. An unallocated agent's utility is $u_i(r) = -t_i(r)$. For the allocated agent i^* , the utility for using the resource at period one is $v_{i^*} - t_{i^*}^{(1)}(r)$ and the utility for not using the resource is $-t_{i^*}^{(0)}(r)$. The rational decision at period 1 is to use the resource if and only if $v_{i^*} - t_{i^*}^{(1)}(r) \geq -t_{i^*}^{(0)}(r)$ (breaking ties in favor of using the resource.) Thus, the expected utility to the allocated agent is

$$u_{i^*}(r) = \mathbb{E} \left[(V_{i^*} - t_{i^*}^{(1)}(r)) \mathbf{1}\{V_{i^*} \geq (t_{i^*}^{(1)}(r) - t_{i^*}^{(0)}(r))\} \right] - t_{i^*}^{(0)}(r) \mathbb{P} \left[V_{i^*} < (t_{i^*}^{(1)}(r) - t_{i^*}^{(0)}(r)) \right] - t_{i^*}(r), \quad (1)$$

where $\mathbf{1}\{\cdot\}$ is the indicator function. Let $r_{-i} \triangleq (r_1, \dots, r_{i-1}, r_{i+1}, \dots, r_n)$.

Definition 1 (Dominant strategy equilibrium (DSE)). A mechanism has a *dominant strategy equilibrium* if for each agent i , all value distribution F_i satisfying (A1)-(A3), there exists a report $r_i^* \in \mathcal{R}$ such that $u_i(r_i^*, r_{-i}) \geq u_i(r_i, r_{-i})$, $\forall r_i \in \mathcal{R}$, $\forall r_{-i} \in \mathcal{R}^{n-1}$.

A direct mechanism, for which the message space is the type space, is *dominant strategy incentive compatible* (DSIC) if truthful reporting of one's type is a DSE. Let $r^* = (r_1^*, \dots, r_n^*)$ denote the report profile in a DSE under mechanism \mathcal{M} .

Definition 2 (Individual rationality (IR)). A mechanism is *individually rational* if for every agent i , all value distribution F_i satisfying (A1)-(A3), $u_i(r_i^*, r_{-i}) \geq 0$, $\forall r_{-i} \in \mathcal{R}^{n-1}$,

In words, IR (or voluntary participation) states that an agent's expected utility is non-negative under her dominant strategy given that she makes rational decisions in period 1 (if allocated), regardless of the reports made by the rest of the agents. We cannot charge unallocated agents without violating IR, thus $t_i(r) \leq 0$ for $i \neq i^*$.

Definition 3 (No deficit (ND)). A mechanism satisfies *no deficit* if, for all possible agent types $\{F_i\}_{i \in N}$ satisfying (A1)-(A3) and all report profile $r \in \mathcal{R}^n$, the expected revenue is non-negative, assuming rational decisions of agents in period 1:

$$\sum_{i \in N} t_i(r) + t_{i^*}^{(0)}(r) \cdot \mathbb{P} \left[V_{i^*} < (t_{i^*}^{(1)}(r) - t_{i^*}^{(0)}(r)) \right] + t_{i^*}^{(1)}(r) \cdot \mathbb{P} \left[V_{i^*} \geq (t_{i^*}^{(1)}(r) - t_{i^*}^{(0)}(r)) \right] \geq 0.$$

A mechanism is *anonymous* if the outcome (assignment, payments) is invariant to permuting the identities of agents. *Deterministic* insists that the outcome is not randomized unless there is a tie. *Always-allocate* requires that the resource is always allocated as long as there is at least one

agent, thus rules out the use of reserve prices. *No-charge* precludes an allocated agent from making a payment to the mechanism if the resource is utilized: $t_{i^*} + t_{i^*}^{(1)} \leq 0$. The objective is to design a mechanisms with desirable properties that maximizes in dominant strategy the *utilization* of the resource, i.e. the probability with which the allocated agent rationally decides to use the resource:

$$ut_{\mathcal{M}}(F) \triangleq \mathbb{P} \left[V_{i^*} \geq (t_{i^*}^{(1)}(r^*) - t_{i^*}^{(0)}(r^*)) \right].$$

3 The CSP Mechanism

We show in this section that under (A1)-(A3), the CSP mechanism achieves higher utilization than the SP auction. The uniqueness and optimality of CSP are discussed in Section 4.

Definition 4 (Contingent Second-Price Mechanism). The *CSP mechanism* collects the bids $b = (b_1, \dots, b_n)$ from the agents.

- Allocation rule: $x_{i^*}(b) = 1$ for $i^* \in \arg \max_{i \in N} b_i$, breaking ties at random.
- Payment rule: $t_{i^*}^{(0)} = \arg \max_{i \neq i^*} b_i$, $t_{i^*}^{(1)}(b) = 0$ and $t_i(b) = 0$ for all $i \neq i^*$

In words, the CSP mechanism collects a single bid b_i from each agent, allocates the resource to the highest bidder in period 0, and charges the second highest bid as no-show penalty if the allocated agent fails to use the resource in period 1.

3.1 Dominant Strategy Equilibrium under CSP

For simplicity, we consider a particular agent and simplify notation, so that V , F , t , $t^{(0)}$, and $t^{(1)}$ denote the random value, type, period 0 payment and contingent payments for this agent. We refactor the payments, and let the “two-part payment” (z, y) be

$$y \triangleq t + t^{(1)}, \text{ and } z \triangleq t^{(0)} - t^{(1)}.$$

Here, y is the “base payment” that an agent makes if she is allocated and chooses to use the resource in period 1, and z is the additional “penalty” that she needs to make if she does not use the resource. The base payment y is always zero under the CSP mechanism.

After learning the realized value v in period 1, an agent gets utility $v - t - t^{(1)} = v - y$ if she uses the resource, and $-t - t^{(0)} = -z - y$ otherwise. A rational agent would decide to use the resource if and only if $v \geq -z$. The base payment y does not affect an agent’s decision in period 1 on whether to use the resource, and the utilization is $\mathbb{P}[V \geq -z]$.

The expected utility (1) for being allocated the right to use the resource as a function of the two-part payment (z, y) can be rewritten as:

$$u(z, y) = \mathbb{E}[V \cdot \mathbb{1}\{V \geq -z\}] - z \cdot \mathbb{P}[V < -z] - y. \quad (2)$$

To economize on notations, we drop the second variable when $y = 0$ and write

$$u(z) \triangleq u(z, 0) = \mathbb{E}[V \cdot \mathbb{1}\{V \geq -z\}] - z \cdot \mathbb{P}[V < -z]. \quad (3)$$

$$= \mathbb{E}[V^+] + \mathbb{E}[V \cdot \mathbb{1}\{-z \leq V < 0\}] - z \cdot \mathbb{P}[V < -z]. \quad (4)$$

It helps to understand the three parts of this expression: $\mathbb{E}[V^+]$ is the highest possible utility that an agent derives from the resource, by utilizing whenever $v \geq 0$ and paying 0 for no-show; $\mathbb{E}[V \cdot \mathbb{1}\{-z \leq V < 0\}]$ is the expected loss of utility when the realized value is negative and the resource is utilized to avoid the payment; and $z \cdot \mathbb{P}[V < -z]$ is the expected amount to pay for not utilizing the resource when the realized value is very low.

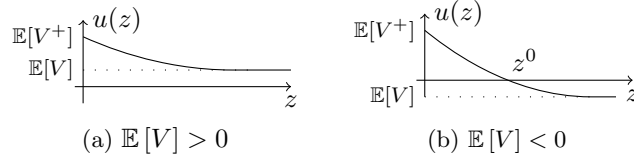


Figure 2: Expected utility for being allocated the resource as a function of penalty z .

Lemma 1. Under (A2), the expected utility $u(z)$ as a function of the penalty z satisfies:

- (i) $u(z) = \mathbb{E}[V^+] - \int_0^z F(-v)dv$,
- (ii) $u(0) = \mathbb{E}[V^+]$, $\lim_{z \rightarrow \infty} u(z) = \mathbb{E}[V]$, and
- (iii) $u(z)$ is continuous and monotonically decreasing w.r.t. z .

See Figure 2. Part (i) of Lemma 1 can be derived by applying integration by parts to (3), and the proof of the rest of the lemma is straightforward. We defer the formal proofs to Appendix A.1. Intuitively, the agent gets the expected positive value $\mathbb{E}[V^+]$ when $z = 0$. As the penalty z increases to infinity, the agent would end up always using the resource and never pays the penalty thus her expected utility converges to $\mathbb{E}[V]$.

Theorem 1 (Dominant Strategy in CSP). *Given assumptions (A1)-(A3), it is a dominant strategy to bid $b_{\text{CSP}}^* = z^0$, the unique zero-crossing of $u(z)$, in the CSP mechanism.*

Proof. From parts (ii) and (iii) of Lemma 1, we know that when $\mathbb{E}[V] < 0$, there is a unique zero crossing z^0 of $u(z)$ s.t. $u(z^0) = 0$, as shown in Figure 2b. In this case, an agent gets non-negative expected utility iff the penalty z is at most z^0 thus z^0 corresponds to the agent's maximum acceptable penalty for no-show. It is then standard for a second price mechanism, noting that the bid sets the maximum penalty the agent will face, that bidding z^0 is a dominant strategy. \square

Without (A3), we have $u(z) > 0$, $\forall z \in \mathbb{R}$, as shown in Figure 2a. This implies that the agent gains in expectation for any penalty z and would always accept such a contract, which we find unreasonable (“pay \$1B if you don’t show up for the spinning class.”) In this case, there is always an incentive to bid higher, and thus no DSE exists.

3.2 Better Utilization than Second Price Auction

The following lemma relates the slope of an agent’s utility functions with utilization.

Lemma 2. While facing a two part payment (z, y) , the utilization, i.e. an agent’s probability of using the resource, corresponds to the derivatives of her expected utility functions w.r.t z :

$$\mathbb{P}[V \geq -z] = 1 - F(-z) = 1 + \frac{d}{dz}u(z) = 1 + \frac{\partial}{\partial z}u(z, y),$$

and is monotonically increasing in z . Moreover, $u(z)$ is convex in z .

The proof is straightforward given part (i) of Lemma 1, the fact that $u(z, y) = u(z) - y$, and that $\frac{d}{dz}u(z) = \mathbb{P}[V \geq -z] - 1$ increases in z . Intuitively, the agent is more likely to use the resource when the penalty is larger. Moreover, the higher the probability that the resource is to be used at penalty z , the less likely the agent is paying the penalty, thus $u(z)$ decreases slower as z increases, which corresponds to the shallower the slope at z .

We now prove that in DSE, the CSP mechanism always achieves higher utilization than the SP auction, which assigns the resource to the highest bidder and charges the second highest bid.

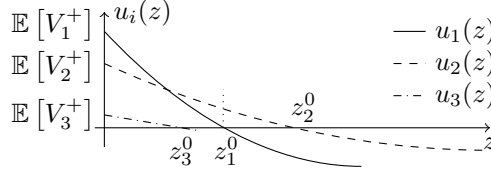


Figure 3: Example economy where the SP winner (agent 1) is different from the CSP winner (agent 2.)

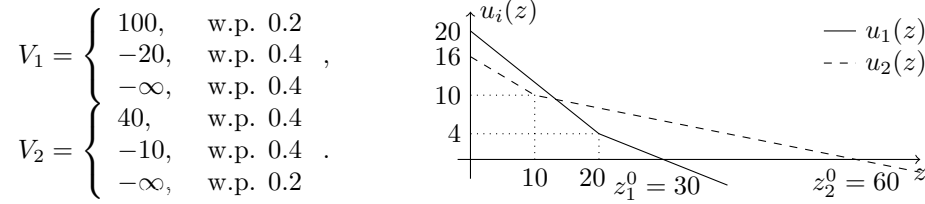


Figure 4: Agents' value distributions and expected utilities in Example 2.

Theorem 2. *For any set of agent types satisfying (A1)-(A3), the CSP mechanism has higher utilization than the SP auction under the dominant strategy equilibria.*

Proof Sketch. The full proof is provided in Appendix A.2. Here we provide the intuition. First, observe that the assigned agent in SP uses the resource as long as $V_{i^*} \geq 0$, achieving utilization $\mathbb{P}[V_{i^*} \geq 0]$. It is a dominant strategy for each agent to bid her expected utility from using the resource: $b_{i,SP}^* = \mathbb{E}[V_i \cdot \mathbb{1}\{V_i \geq 0\}] = \mathbb{E}[V_i^+]$. Consider the following two cases:

Case 1: SP and CSP allocate the resource to the same agent (say agent 1), the utilization under CSP $\mathbb{P}[V_1 \geq -z]$ is always (weakly) higher than that under SP $\mathbb{P}[V_1 \geq 0]$, for any penalty $z \geq 0$.

Case 2: SP and CSP allocate the resource to agent 1 and 2 respectively. Ignoring ties, we must have $\mathbb{E}[V_1^+] > \mathbb{E}[V_2^+]$, and $z_1^0 < z_2^0$, as shown in Figure 3. This implies $u_1(0) - u_1(z_1^0) > u_2(0) - u_2(z_1^0)$. In order for this to happen, the slope of $u_2(z)$ at any penalty level $z \in [z_1^0, z_2^0]$ (where the CSP payment resides) must be shallower than that of $u_1(z)$ at $z = 0$, due to the convexity of the expected utility functions. This translates to CSP having strictly higher utilization than SP given Lemma 2. \square

The following examples illustrate the improvement in utilization from CSP over SP, and show that SP can be arbitrarily worse than CSP.

Example 2 (Double gain in CSP). Consider two agents with value distributions and expected utilities as shown in Figure 4. Compared with Agent 2, Agent 1 has higher value for the facility, but lower probability of willing to use the resource and higher probability for a hard constraint. Under SP, $b_{SP,1}^* = 20, b_{SP,2}^* = 16$ thus Agent 1 is allocated and the utilization is $\mathbb{P}[V_1 \geq 0] = 0.2$. Whereas under CSP $b_{CSP,1}^* = z_1^0 = 30$ and $b_{CSP,2}^* = z_2^0 = 60$. Agent 2 is allocated and charged penalty $b_{CSP,1}^* = 30$ for no-show, thus the utilization is $\mathbb{P}[V_2 \geq -z_1^0] = 0.8$. This is higher than $\mathbb{P}[V_2 \geq 0] = 0.4$, the probability with which the resource is utilized, if Agent 2 is allocated the resource under SP in some other economy.

Example 3 (SP arbitrarily worse). Under the (w_i, p_i) model introduced in Example 1, the expected utility for agent i given penalty z is $u_i(z) = w_i p_i - (1 - p_i)z$. It is a dominant strategy to bid $b_{i,SP}^* =$

$\mathbb{E}[V_i^+] = w_i p_i$ under SP and the zero-crossing $b_{i,\text{CSP}}^* = z_i^0 = w_i p_i / (1 - p_i)$ under CSP. Consider an economy where there are two agents with types: $p_1 = \varepsilon$, $w_1 = 1/\varepsilon$; and $p_2 = 1 - \varepsilon$, $w_2 = 1$ for some $\varepsilon > 0$ very small. We can verify that agent 1 is allocated under SP since $b_{1,\text{SP}}^* = 1 > b_{2,\text{SP}}^* = 1 - \varepsilon$ whereas agent 2 is allocated under CSP since $b_{2,\text{CSP}}^* = (1 - \varepsilon)/\varepsilon > 1/(1 - \varepsilon) = b_{1,\text{CSP}}^*$. The utilization under SP and CSP is ε and $1 - \varepsilon$, respectively; thus, CSP can have arbitrarily better utilization than SP by selecting a better winner.

We see that CSP's higher utilization comes from two different aspects of its design:

- Charging a penalty $z > 0$ changes the period 1 decision of the allocated agent, promoting the resource to be used with higher probability.
- Agents with higher probability of showing up have utility functions that decrease more slowly with penalty, thus have relatively higher zero-crossing points and bid higher in CSP

One might wonder about a comparison between slightly more general mechanisms. Consider, for example, introducing a reserve price $R > 0$ into the CSP mechanism, where the resource is only allocated when $b_{i^*} \geq R$ for $i^* \in \arg \max_{i \in N} b_i$, and the allocated agent is charged $t_{i^*}^{(0)} = \max(\max_{i \neq i^*} b_i, R)$ as penalty. We call this the *CSP+R mechanism*. Under (A1)-(A3), it remains a dominant strategy for agents to bid the zero-crossings $b_{i,\text{CSP+R}}^* = z_i^0$ under CSP+R. Similarly, the SP auction can be modified to charge an additional fixed penalty $C > 0$ (termed the *SP+C mechanism*), which introduces a penalty to the assigned agent in the event that the resource goes wasted. We can show that when the reserve penalty R is set to be equal to the fixed penalty C , utilization under CSP+R dominates that of the SP+C mechanism profile-by-profile (see Theorem 9 in Appendix A.7.) We discuss the effect of reserve prices in more detail in Sections 4 and 6.

4 Characterization and Optimality of CSP

In this section, we consider the full space of mechanisms, including those with two-part payments that use both the base payment y and a no show penalty z . We consider the optimal mechanism design problem under (some subset) of the following properties:

- | | |
|-----------------------------------|--|
| P1. Dominant-strategy equilibrium | P5. Deterministic (unless breaking ties) |
| P2. Individually rational | P6. Always-allocates the resource |
| P3. No deficit | P7. No charge |
| P4. Anonymous | |

We prove that when the type space is the set of all value distributions satisfying (A1)-(A3), the CSP mechanism is unique amongst mechanisms with properties (P1)-(P7). For this, we make use of a key lemma (Lemma 3), which characterizes the set of all possible outcomes under mechanisms that satisfy (P1)-(P5). Combined with (P6) and (P7), we can show that CSP is unique. We also prove that the utilization of the CSP mechanism is not dominated by any mechanism in the broader class of mechanisms that satisfy properties (P1)-(P5), and that the CSP mechanism is optimal for agents with the simple (w_i, p_i) types from Example 1. Moreover, we prove that the CSP mechanism maximizes utilization among a broad subclass of mechanisms satisfying (P1)-(P6), and that relaxing (P6) by adding reserve prices rarely improves utilization.

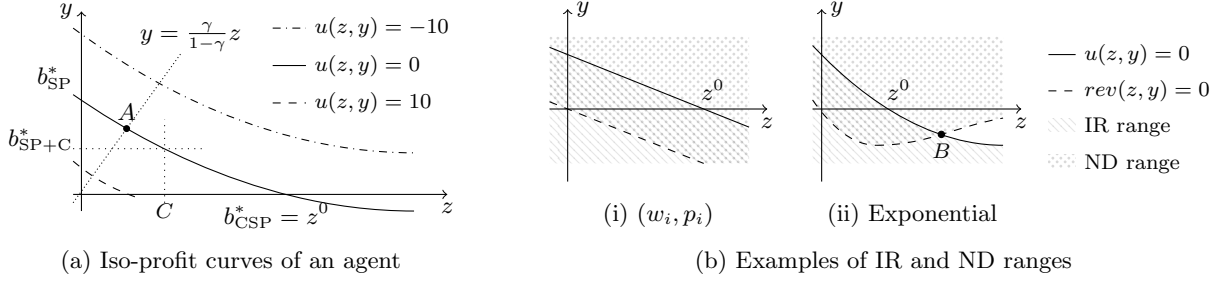


Figure 5: Illustration of iso-profit curves, IR and ND ranges in the 2-D payment space.

4.1 Utilities in The Two-Dimensional Payment Space

For the formal analysis of uniqueness and optimality, we appeal to the standard revelation principle from mechanism design, and consider the space of direct-revelation mechanisms.

Zero-profit Curves Recall that in the CSP mechanism the base payment $y = t + t^{(1)}$ is always zero. To study general mechanisms, we move back to the full (z, y) payment space, and work with *iso-profit curves* which are sets of (z, y) pairs for which $u(z, y) = c$ for some constant c , i.e. an agent will be indifferent to all payments (z, y) that reside on the same iso-profit curve. See Figure 5a.

Because $u(z, y) = u(z) - y$ (see (2) and (3)), the *zero-profit curve* (i.e. where $c = 0$, the solid line depicted in Figure 5a) is characterized by $\{(z, y) \mid y = u(z)\}$. This has the same shape as the expected utility function $u(z)$ in Figure 2 and is continuously decreasing and convex according to Lemmas 1 and 2. Observing $\frac{\partial}{\partial y}u(z, y) = -1$, we know that other iso-profit curves are vertical shifts of the zero-profit curve, and recall from Lemma 2 that the utilization for an agent facing payments (z, y) directly relates utilization to the slope of the zero-profit curve at point (z, y) : $\mathbb{P}[V \geq -z] = 1 + \frac{d}{dz}u(z) = 1 + \frac{\partial}{\partial z}u(z, y)$.

IR and ND Ranges If an allocated agent is charged payment (z, y) above her zero-profit curve, her expected utility would be negative since $u(z, y)$ decreases in z and y . We call the area weakly below an agent's zero-profit curve the *IR-range* for the agent. For any two-part payment (z, y) , the expected revenue (e.g. payment from the agent to the mechanism) is $rev(z, y) = y + z \cdot \mathbb{P}[V < -z]$. We call the set of payments (z, y) for which $rev(z, y)$ is non-negative as the *ND-range* (no-deficit range) for this agent. The ND range depends an agent's own type, meaning that fixing two part payment (z, y) , whether the mechanism runs into deficit depends on the type of the assigned agent.

Example 4. Consider the (w_i, p_i) model (Example 1.) The zero-profit curve is characterized by $\{(y, z) \in \mathbb{R}^2 \mid y = u(z) = w_i p_i - (1 - p_i)z\}$. The expected revenue that the mechanism collects is $rev(z, y) = y + (1 - p_i)z$, thus the ND range is lower-bounded by $y = -(1 - p_i)z$. See Figure 5b(i).

Example 5 (Exponential model). Under the exponential model, the utility for agent i to use the resource is a fixed value $w_i > 0$ minus a random opportunity cost, which is exponentially distributed with parameter $\lambda_i > 0$, as illustrated in Figure 6. The expected value $\mathbb{E}[V_i] = w_i - 1/\lambda_i$ where $1/\lambda_i$ is the expected value of the opportunity cost. With $w_i < 1/\lambda_i$, assumptions (A1)-(A3) are satisfied. For an agent with an exponential value distribution, the zero-profit curve, IR and ND ranges are as shown in Figure 5b(ii) (see derivations in Appendix B.)

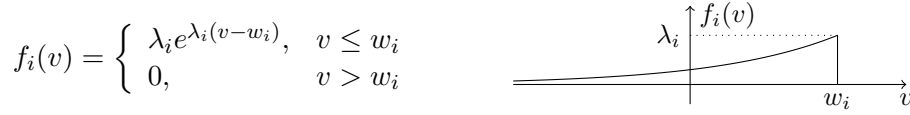


Figure 6: Agent value distribution in the exponential type model.

The First Best Under any IR and ND mechanism, the payment facing the assigned agent must reside in the intersection of the IR and ND ranges of the assigned agent. Given an agent's type, we can compute the highest utilization induced by any two-part payment in this intersection, which we call the *first-best* utilization of this agent. For the exponential type, the first-best is achieved by charging the agent (z_B, y_B) at point B in Figure 5b(ii), which is the two-part payment with the highest penalty component in the IR and ND range. For the (w_i, p_i) type, though there is no upper bound on the highest IR and ND penalty, the utilization is always p_i , and is not affected by z .

DSE Bids The mechanisms that we have discussed so far restrict the payments to an ordered, one-dimensional subspace of the (z, y) payment space: CSP considers $\{(z, y) | y = 0\}$ where all agents prefer a smaller z , and SP restricts $\{(z, y) | z = 0\}$ where all agents prefer a smaller y . The crossing point of the an agent's zero-profit curve and the one-dimensional payment space determines the agent's maximum willingness to pay under each mechanism, thus translates into the agent's DSE bids in these second price mechanisms. Under CSP, the DSE bid $b_{i,\text{CSP}}^* = z_i^0$ corresponds to the crossing point of $y = 0$ and the agent's zero-profit curve, and under SP, the DSE bids $b_{i,\text{SP}}^* = \mathbb{E}[V_i^+]$ correspond to the crossing points of the zero-profit curve with $z = 0$. See Figure 5a.

To see another example, consider a generalization of CSP that collects the second highest bid as the no-show penalty and also charges a γ -fraction of the second highest bid as the base payment.

Definition 5 (The γ -CSP Mechanism). The γ -CSP mechanism, parametrized by $\gamma \in [0, 1]$, collects a single bid from each agent.

- Allocation rule: $x_{i^*}(b) = 1$ for $i^* \in \arg \max_{i \in N} b_i$, breaking ties at random.
- Payment rule: $t_{i^*}^{(0)} = \arg \max_{i \neq i^*} b_i$, $t_{i^*}^{(1)}(b) = \gamma \cdot t_{i^*}^{(0)}$ and $t_i(b) = 0$ for all $i \neq i^*$

Payments reside on $\{(z, y) | y = \gamma/(1 - \gamma)z\}$, and the SP ($\gamma = 1$) and CSP ($\gamma = 0$) are special cases. We prove that it is a dominant strategy to bid the sum of the two coordinates of the crossing point between the zero-profit curve and $y = \frac{\gamma}{1-\gamma}z$ (i.e. $b_i^* = z_A + y_A$ as in Figure 5a; see Theorem 7 in Appendix A.3.) We return to this mechanism below.

4.2 Uniqueness and Optimality of CSP

Define the *frontier* of an economy N to be the upper-envelope of the zero-profit curves of all agents, i.e., $\{(z, y) | y = \bar{u}_N(z)\}$ where $\bar{u}_N(z) \triangleq \max_{i \in N} u_i(z)$. This characterizes the maximum willingness to pay by all agents in N . As the upper envelope of a finite set of decreasing convex functions, $\bar{u}_N(z)$ is also decreasing and convex, and has a unique zero-crossing, which we denote as z_N^0 . Define the frontier of the sub-economy without agent i as $\bar{u}_{N \setminus \{i\}}(z) \triangleq \max_{j \neq i} u_j(z)$, and the m^{th} frontier of the economy as the m^{th} upper envelope of $\{u_i(z)\}_{i \in N}$. See Figure 7.

The following key lemma characterizes possible outcomes for mechanisms satisfying (P1)-(P5).

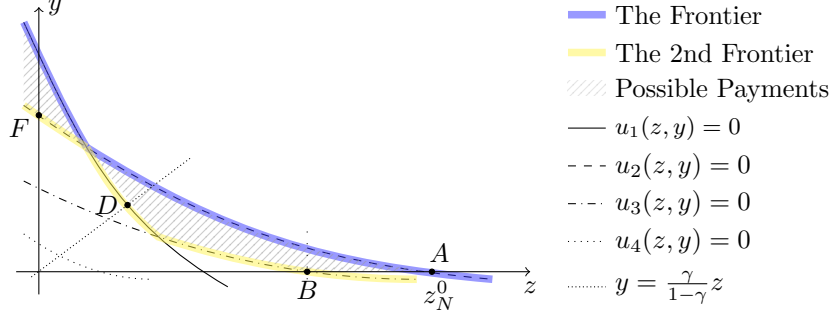


Figure 7: Characterization of possible outcomes for mechanisms under (P1)-(P5).

Lemma 3. Assume the type space is the set of all value distributions satisfying (A1)-(A3). Fix any mechanism that satisfies (P1)-(P5), the allocated agent i^* and the two-part payment (z^*, y^*) the allocated agent i^* is charged:

- (i) (z^*, y^*) resides weakly below $u_{i^*}(z)$;
- (ii) $u_{i^*}(z)$ must be a part of the frontier $u_N(z)$ of all agents;
- (iii) (z^*, y^*) resides weakly above the frontier of the rest of the economy $u_{N \setminus \{i^*\}}(z)$;
- (iv) The allocated agent faces a non-negative base payment $y^* \geq 0$;
- (v) The utilization corresponds to the slope of the frontier at z^* : $ut_{\mathcal{M}}(F) = \frac{d}{dz} \bar{u}_N(z)|_{z=z^*} + 1$, which increases in z^* and attains its maximum at $z^* = z_N^0$.

Proof Sketch. We defer the full proof to Appendix A.4 and provide intuitions here. (i) is implied by IR. (ii) and (iii) require that the allocated agent i^* to be on the frontier of all agents, and that (z^*, y^*) resides in between the first and second frontiers, as illustrated in Figure 7. Otherwise, there exists $i \neq i^*$ that strictly prefers the winner's outcome of "getting allocated and pay (z^*, y^*) " to her outcome "not getting allocated", which enables us to construct an economy where DSIC is violated.

(iv) implies i^* cannot be paid a bonus for showing up. Assume otherwise, that i^* with type u_{i^*} is allocated and charged payment (z^*, y^*) with $y^* < 0$. We can construct a type u'_{i^*} of (w_i, p_i) model s.t. (z^*, y^*) is inside u'_{i^*} 's IR range but outside of her ND range. Replace u_{i^*} with u'_{i^*} in the original economy, we can show that either ND is violated, or the agent with type u'_{i^*} has a useful deviation.

For part (v), note that (i)-(iii) imply $u_{i^*}(z) = \bar{u}_N(z)$ must hold for a small neighborhood around z^* , therefore the utilization, which corresponds to the slope of $u_{i^*}(z)$ at z^* also corresponds to the slope of the frontier: $\mathbb{P}[V_{i^*} \geq z^*] = \frac{d}{dz} u_{i^*}(z) + 1|_{z=z^*} = \frac{d}{dz} \bar{u}_N(z)|_{z=z^*} + 1$. Since $u_N(z)$ is convex, the utilization $\frac{d}{dz} \bar{u}_N(z)|_{z=z^*} + 1$ increases z^* . The highest possible utilization under (P1)-(P5) is achieved when the penalty is $z^* = z_N^0$, the highest penalty in the range of possible payments determined by (i)-(iv). In this case, the agent with the highest zero-crossing must be allocated. \square

With this characterization, we prove the uniqueness of CSP given (P1)-(P7).

Theorem 3. Assume the type space is the set of all value distributions satisfying (A1)-(A3). Then CSP is unique: it is the only mechanism with properties (P1)-(P7).

We defer the full proof to Appendix A.5 due to the space limit. For some intuition, given (P7) and Lemma 3, the payment facing the allocated agent must be consisted of a penalty in between the second and highest zero-crossings, and no base payment. In this case, the agent with the highest zero-crossing z_i^0 (i.e. the CSP winner) must be allocated since IR is violated otherwise. (P6) further pins down the penalty to be exactly the second highest zero-crossing (i.e. the CSP payment.) If not, either the allocated agent has a useful deviation, or in some other economy the resource would not be allocated. We also prove that CSP optimal for (w_i, p_i) model given (P1)-(P5).

Theorem 4. *Assume the type space is the set of all (w_i, p_i) value distributions. CSP achieves highest possible utilization among mechanisms satisfying (P1)-(P5), type profile by type profile.*

This follows because, by part (v) of Lemma 3, we know that the only way to beat the CSP mechanism is to charge the CSP winner a penalty higher than the second highest zero-crossing (this would incentivize higher utilization). But penalties do not improve the utilization in the (w_i, p_i) model because if the agent cannot use the resource, she cannot whatever the penalty. Another implication of Lemma 3 is that CSP is not dominated given (P1)-(P5).

Theorem 5. *Assume the type space is the set of all value distributions satisfying (A1)-(A3). No mechanism under (P1)-(P5) achieves weakly higher utilization than CSP for all economies, and a strictly higher utilization than CSP for at least one economy.*

See Appendix A.5 for the proof. Intuitively, if a mechanism charges the allocated agent a higher penalty than the second highest zero-crossing in some economy, in some other economy it must leave the resource unallocated or charge a smaller penalty, resulting in lower utilization than CSP.

4.3 Tightness of the Result

CSP is unique given (P1)-(P7), and is undominated given (P1)-(P5). In regard to tightness of the results, note that relaxing any one of (P1)-(P3) respectively, we achieve higher utilization than CSP for all economies by (P1) implementing the full information first best, (P2) charging the allocated agent infinite no-show penalty, (P3) paying the allocated agent infinite bonus for showing up. Relaxing (P4) by always allocating to the same agent or (P5) allocating uniformly at random may achieve higher utilization than CSP for some economies, but results in lower utilization on average.

Relaxing either (P6) or (P7), CSP is no longer unique or optimal profile by profile. On the other hand, the performance of CSP should be expected to not only be undominated but good on average.

Relaxing (P6) and imposing (P7), we may set reserve penalties which improves utilization for some economies. Simulation results, however, show that reserve penalties decrease average utilization for simple type distributions (see Section 6.) The reason why reserve prices improve revenue in second price auctions but not utilization is that utilization given penalty z consists of two parts:

$$ut_{i^*}(z) = \mathbb{P}[V_{i^*} \geq z] = \mathbb{P}[V_{i^*} \geq 0] + \mathbb{P}[-z \leq V_{i^*} < 0]. \quad (5)$$

If an item is not sold because of a small reserve price, the loss of revenue is small and bounded by the reserve. In contrast, if a resource is not allocated, the loss of utilization may not be small, due to the loss of the first part of (5): $\mathbb{P}[V_{i^*} \geq 0]$, the probability with which the agent uses the resource if she is given the resource for free. In fact, we prove that for economies with a single agent and simple type distributions, the optimal reserve penalty is zero (see Appendix A.6.)

Imposing (P7) “no charge” may seem without loss— if an allocated agent i^* is charged a positive base payment $y^* > 0$, reducing y^* to zero, increasing z^* while keeping $u_{i^*}(z^*, y^*)$ the same, utilization is (weakly) improved without decreasing the utility of agent i^* . This intuition seems to be supported by the result that the γ -CSP mechanism family, which violates (P7), is still dominated by CSP.

Corollary 1. *For any set of agent types satisfying (A1)-(A3), CSP achieves weakly higher utilization than the γ -CSP mechanism for any $\gamma \in (0, 1]$.*

However, CSP does not dominate all mechanisms that satisfy (P1)-(P6) profile-by-profile. This is because with the multi-dimensional types and the (z, y) payment space, different types disagree on which two-part payment is more favorable due to different probabilities of paying the penalty. This disagreement enables mechanisms to select the (z, y) contract as a function of an agent’s *own* report without violating DSIC, and to sometimes charge a higher penalty than the CSP payment. We have experimented with such a class of mechanisms, but have not found them to achieve higher average utilization than CSP in simulation (see Appendix E)

5 Assignment of Multiple Resources

In this section, we generalize the model to allow for assigning multiple resources but where each agent remains interested in receiving an assignment of at most one resource (i.e., the unit-demand model). We propose two DSIC mechanisms, the contingent $(m+1)^{\text{th}}$ price mechanism for assigning m identical resources (e.g., spots in a spinning class,) and the generalized CSP mechanism for assigning m heterogeneous resources (e.g., time slots of a charging station.)

5.1 Multiple Identical Resources

Consider first the scenario of assigning m identical resources. An agent’s type is still F_i , describing her value distribution for using one unit. The CSP mechanism can be generalized as a $(m+1)^{\text{th}}$ price mechanism, which collects a single bid from each agent, allocates to the m highest bidders, and charge the $(m+1)^{\text{th}}$ highest bid as the no-show penalty:

Definition 6 (Contingent $(m+1)^{\text{th}}$ Price Mechanism). Let $b = (b_1, \dots, b_n)$ be a bid profile and w.l.o.g. reorder the agents as $b_1 \geq b_2 \geq \dots \geq b_n$, breaking ties at random.

- Allocation rule: $x_i(b) = 1$ for $i \leq m$ and $x_i(b) = 0$, otherwise.
- Payment rule: $t_i^{(0)}(b) = b_{m+1}$ and $t_i^{(1)}(b) = 0$ for $i \leq m$; $t_i(b) = 0$ for all i .

It remains a dominant strategy for an agent to bid her zero-crossing z_i^0 under the contingent $(m+1)^{\text{th}}$ price mechanism. The CSP+R, γ -CSP and SP mechanisms can also be generalized similarly, and the dominant strategies remain the same as in the allocation of a single resource.

Theorem 6. *Assume the type space is the set of all value distributions satisfying (A1)-(A3). For allocating m identical resources, the contingent $(m+1)^{\text{th}}$ price mechanism is the only mechanism that satisfy (P1)-(P7). Moreover, this mechanism is optimal for the (w_i, p_i) type model under (P1)-(P5), and dominates the generalization of the γ -CSP mechanisms for all $\gamma \in (0, 1]$ in utilization.*

In particular, the contingent $(m+1)^{\text{th}}$ price mechanism dominates the $(m+1)^{\text{th}}$ price auction (i.e., the generalization of SP.) Similar to Lemma 3, we can show that the allocated agents must be on the top m upper envelopes of the economy, the payment must be weakly above the $(m+1)^{\text{th}}$

envelope, and the summation of the slopes of the top m upper envelopes, which corresponds to the total utilization, increases monotonically as the penalty increases. The rest of the proof follows the same arguments as those of Theorem 3 and Corollary 1.

5.2 Multiple Heterogeneous Resources

Consider now the problem of assigning m different resources. Let $N = \{1, 2, \dots, n\}$ be the set of agents and $M = \{a, b, \dots, m\}$ be the set of resources. For each $a \in M$, the value for each agent i to use resource a is a random variable $V_{i,a}$ with CDF $F_{i,a}$. We assume $\{V_{i,a}\}_{i \in N, a \in M}$ are independent, and that for each $i \in N$, $a \in M$, $F_{i,a}$ satisfies (A1)-(A3). $\{F_{i,a}\}_{a \in M}$ corresponds to agent i 's type.

This is the utilization-maximization version of the classical unit-demand assignment problem. The SP mechanism can be generalized as the VCG mechanism [27, 8, 16], which collects a single bid from each agent for each alternative $b_{i,a}$, assigns the resources to maximizes the summation of total bids, and charges each assigned agent in period 0 the negative externality (in terms of bids) that she imposes on the rest of the agents. Since expected utility (1) is quasi-linear in the period 0 payment t_i , bidding one's true values $b_{i,a}^* = \mathbb{E}[V_{i,a}^+]$.

What is interesting is that the naïve generalization of CSP, which collects bids and charges the allocated agents the VCG payments as penalties, fails to be incentive compatible because $u_{i,a}(z)$ as in (3) is not quasi-linear in the penalty z . In this way, we see a surprising connection with the literature on DSIC assignment under non quasi-linear utilities. We generalize the CSP mechanism by setting the contingent payments for each resource to be the minimum competitive equilibrium (CE) prices [12, 1]. A set of penalties $\{z_a\}_{a \in M}$ is a set of CE prices if when resources are assigned to agents in an agent-maximizing manner given these penalties, no resource is assigned more than once, and unassigned resources have penalty zero.

Definition 7 (Generalized CSP Mechanism (GCSP)). The GCSP mechanism collects from agents value distributions $\{F_{i,a}\}_{i \in N, a \in M}$, computes the expected utilities as functions of no-show penalties $\{u_{i,a}(z)\}_{i \in N, a \in M}$ according to (3) and the minimum competitive equilibrium penalties $\{z_a\}_{a \in M}$.

- Allocation rule: assign to each agent her most preferred resource given $\{z_a\}_{a \in M}$: $x_i = a_i^* = \arg \max_{a \in M} u_{i,a}(z_a)$ if $\max_{a \in M} u_{i,a}(z_a) > 0$, breaking ties to clear the market.
- Payment rule: charge each allocated agent $t_i^{(0)} = z_{a_i^*}$ as no-show penalty if the agent is allocated some resource $a_i^* \in M$. All other payments are zero.

Demange and Gale [12] prove that for the unit-demand multi-item assignment problem where agents' utilities are non-quasi-linear but decrease continuously in payments, as is the case for $\{u_{i,a}(z)\}_{i \in N, a \in M}$, the minimum CE prices exist. Moreover, the mechanism which sets the minimum CE prices and assigns the items in an agent-maximizing manner, is truthful, and clears the market. Thus we conclude that the generalized CSP mechanism is well defined, and that truthful reporting is a DSE. For the computation of the CE prices, Alaei et. al. [1] provide a recursive algorithm that computes prices based on outcomes in economies with less agents and less alternatives.

For assigning m identical resources, the minimum CE penalties are exactly the $m + 1^{\text{th}}$ highest zero-crossing among all agents, thus GCSP coincides with the contingent $m + 1^{\text{th}}$ price mechanisms and generalizes both that and the CSP mechanism.

Unlike the assignment of identical resources, the utilization under GCSP need not dominate that of the VCG mechanism. We show this through a pair of examples. Still, simulation results in Section 6 show that the GCSP mechanism almost always achieves higher utilization than VCG.

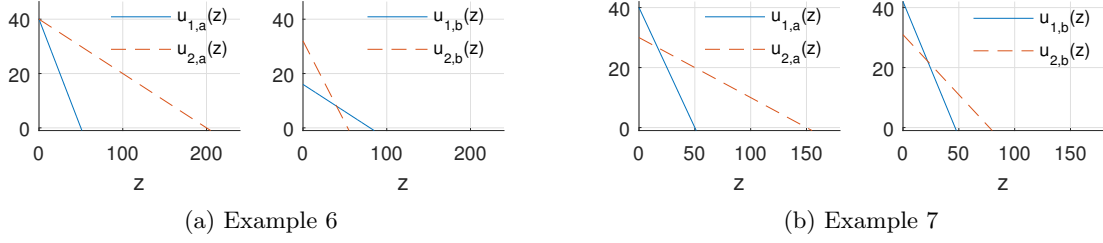


Figure 8: An example of utility curves for the heterogeneous assignment problem.

Example 6 (GCSP beats VCG). Consider an economy two resources a, b and two agents 1 and 2 and the following types according to the (w_i, p_i) type model:

- Agent 1: $w_{1,a} = 200$, $p_{1,a} = 0.2$; $w_{1,b} = 20$, $p_{1,b} = 0.8$.
- Agent 2: $w_{2,a} = 50$, $p_{2,a} = 0.8$; $w_{2,b} = 80$, $p_{2,b} = 0.4$.

The expected utilities are as shown in Figure 8. Under VCG, agents bid $b_{i,j}^* = w_{i,j}p_{i,j}$. Agent 1 is allocated resource a and agent 2 is allocated resource b , and the total utilization is $p_{1,a} + p_{2,b} = 0.6$.

The lowest CE penalties can be computed as $z_a = 30$ and $z_b = 0$. Under these penalties, $u_{1,a}(z_a) = w_{1,a}p_{1,a} - (1 - p_{1,a})z_a = 16$, $u_{1,b}(z_b) = w_{1,b}p_{1,b} - (1 - p_{1,b})z_a = 16$ thus agent 1 is indifferent between the two resources. For agent 2, $u_{2,a}(z_a) = w_{2,a}p_{2,a} - (1 - p_{2,a})z_a = 40 - 0.2 \cdot 30 = 34$, $u_{2,b}(z_b) = w_{2,b}p_{2,b} - (1 - p_{2,b})z_b = 32$, thus agent 2 strictly prefers resource a . Under the GCSP mechanism, agent 1 is allocated resource b , while agent b is allocated resource a . The total utilization under GCSP is therefore $p_{1,b} + p_{2,a} = 1.6$, higher than that of VCG.

We can see from Example 6 that comparing with the VCG mechanism, which favors agents with high vertical intercept, agents with high zero-crossings and therefore shallower expected utility functions are more likely to get allocated in GCSP. This can lead in turn to higher utilization. For agents with continuous types (e.g. the exponential type model,) utilization is further improved due to the penalty charged by the mechanism. Still, VCG may provide higher utilization in somewhat extreme examples where the number of agents is small and there is little competition.

Example 7 (VCG beats GSCP). Consider an economy with the following two agents 1 and 2, two items a and b , the $w_i p_i$ type model, and agent types:

- Agent 1: $v_{1,a} = 200$, $p_{1,a} = 0.2$; $v_{1,b} = 550$, $p_{1,b} = 0.1$.
- Agent 2: $v_{2,a} = 37.5$, $p_{2,a} = 0.8$; $v_{2,b} = 66.67$, $p_{2,b} = 0.6$.

The expected utility curves are as shown in Figure 8b. Under VCG, agent 1 gets resource b and pays 10, agent 2 gets resource a and pays 0. Total utilization is $0.1 + 0.8 = 0.9$. Under GCSP, we can compute following the same steps as in Example 6 that agent 1 gets resource a and pays 0, agent 2 gets resource b and pays 16.67 for no show penalty. The total utilization is $0.2 + 0.6 = 0.8$, which is lower than that of VCG.

6 Simulation Results

In this section, we compare resource utilization for different mechanisms under the natural exponential type model (Example 5,) where agents' uncertain values are parametrized by a fixed value

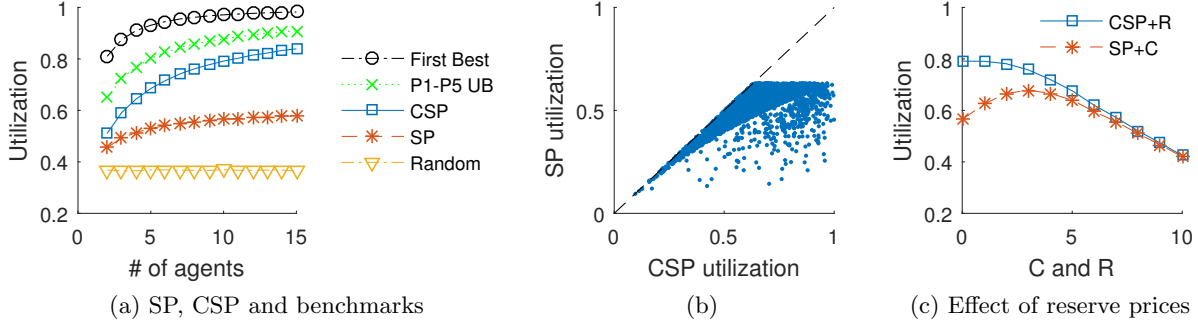


Figure 9: Utilization of a single resource under the exponential model

and the expected opportunity cost. A type distribution specifies the distributions of these parameters. The simulation results are very robust to different type models and different distributions on types.²

6.1 Single Resource

For allocating a single resource, an agent's value for using the resource is $V_i = w_i - O_i$ where $w_i > 0$ is the fixed value and $O_i \sim \text{Exp}(\lambda_i)$ is the opportunity cost. $\mathbb{E}[V_i] = w_i - \lambda_i^{-1}$ where λ_i^{-1} is the expected value of the opportunity cost. We consider the type distribution where the value and the expected opportunity cost λ_i^{-1} are uniformly distributed: $\lambda_i^{-1} \sim U[0, L]$ and $w_i \sim U[0, \lambda_i^{-1}]$. The results are robust to the range L , and $w_i < \lambda_i^{-1}$ guarantees (A1)-(A3) are satisfied.

SP, CSP and Benchmarks We set $L = 10$, vary the number of agents from 2 to 15, and consider the average utilization over 10,000 randomly generated profiles under SP, CSP and other benchmarks. See Figure 9a. The *First Best* is the highest achievable utilization subject to the IR and ND as discussed in Section 4. *P1-P5 UB* is the upper bound on utilization under (P1)-(P5) as discussed in Lemma 3. *Random* is the average utilization achieved by randomly allocating the resource to one of the agents without charging any payment. The CSP mechanism achieves a large improvement compared with the SP auction and Random, and performs relatively well when compared with the P1-P5 UB. With three agents, Figure 9b compares the utilization under CSP and SP profile-by-profile. The utilization of CSP dominates that of SP, as proved in Theorem 2.

Fixed Penalty and Reserve Prices Here, we fix the number of agents to be $n = 10$. Figure 9c examines utilization under CSP with reserve price (CSP+R) and under SP with fixed penalty (SP+C). We can see that while $C > 0$ may improve the utilization of SP, it is not enough to achieve higher utilization than CSP. Fixing $R = C$, the utilization of CSP+R not only achieves a higher average utilization than SP+C but dominates that of SP+C profile-by-profile (as proved in Theorem 9, not shown in the figure.) We see that CSP+R is best with zero reserve penalty. This means that the improvement in utilization due to an additional penalty is smaller than the loss of utilization due to the resource becoming unallocated (see discussions in Section 4.3.)

²See Appendices B and C for derivation of bids in different models, and for a large sweep of simulation results.

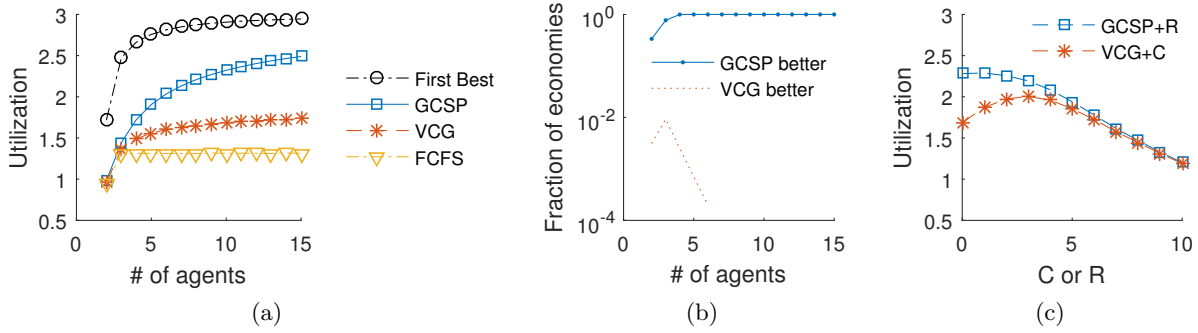


Figure 10: Utilization for assigning three resources under the exponential type model.

6.2 Multiple Resources

In this section, we compare the performance of different mechanisms when the resources are not identical. For each agent i and alternative a , we model $V_{i,a}$ as being an exponential type with parameters $w_{i,a}$ and $\lambda_{i,a}$, and we assume the opportunity costs $\lambda_{i,a}^{-1}$ are i.i.d. $\lambda_{i,a}^{-1} \sim U[0, L]$ and that given $\lambda_{i,a}$, the value $w_{i,a} \sim U[0, \lambda_{i,a}^{-1}]$.

We fix the number of resources to be $m = 3$, and the maximum opportunity cost $L = 10$. The results are robust to different m and L . The average utilization over 10,000 randomly generated profiles is shown in Figure 10a, as we vary the number of agents from 2 to 15. The *First Best* benchmark computes for each economy the assignment that maximizes total utilization under the full information first best, and *FCFS* (first come first serve) is equivalent to the random serial dictatorship, where agents arrive in random order and gets assigned for free her favorite resource that is still available.

As the number of agents increases, the utilization of GCSP increases significantly because competition results in agents with higher probability of utilization being selected and increases the penalties which further boost utilization. Unlike in the single resource case, the utilization under GCSP does not dominate that of VCG profile-by-profile. The fraction of profiles for which GCSP has strictly higher utilization, and for which VCG has strictly higher utilization are shown in Figure 10b. When the number of agents is very small, for about 1% of the profiles, VCG has strictly higher utilization than GCSP. With three or more agents, GCSP is almost always better.

As with the SP auction, the VCG mechanism for assigning multiple resources can also be generalized to include a fixed penalty C (termed VCG+C). The effect of reserve prices R for GCSP (the GCSP+R) and fixed penalty C for VCG+C is shown in Figure 10c, where the number of agents n is fixed to be 10 and the reserve R and penalty C vary from 0 to 10. There is a very small improvement in utilization for the GCSP+R mechanism comparing with no reserve when R is close to zero, otherwise the patterns are similar as in the case of single resource assignment.

7 Conclusion

We introduced the problem of maximizing resource utilization in a setting where a plan needs to be made but there is uncertainty about the actual values of agents that will be realized for resources (and indeed, these may be negative, reflecting that they have better things to do!). For allocating a single resource, we proved that the natural contingent second-price mechanism is unique under a set of desired criteria, and maximizes utilization across a larger class of mechanisms. We

also generalized the mechanism to assign multiple resources, connecting this problem with that of assignment under non quasi-linear utility, and giving theoretical results for the case of multiple, identical items, and strong support from simulation for the case of multiple, heterogeneous items.

Directions for future work include weakening the assumptions on the type space for the optimality of CSP, developing theoretical results for assigning multiple, heterogeneous resources, and designing contingent mechanisms to incentivize utilization or reliability in repeated assignments of resources or tasks (e.g. scheduling serving times at a childcare co-op operated by the parents, rebalancing in bike-sharing systems by the customers.) More generally, we see a rich agenda in the design of what we think about as “coordination mechanisms”—these are mechanisms that need to form a plan and construct payment schedules to promote coordinated outcomes by a system of agents. Exploring the effect of present-bias [14] in the coordination of future events and designing commitment devices [5] through contingent-payment mechanisms is another interesting direction.

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Appendix

We provide missing proofs, theorems and discussions in Appendix A. Derivation of expected utility functions, DSE bids and bid distributions for simple type model and type distributions are provided in Appendix B, followed by additional simulation results for single or multi-resource assignment in Appendix C.

The welfare interpretation of our problem and the optimality of the SP+C mechanism are discussed in in Appendix D. We provide more details on the two-dimensional payment space and agents heterogeneous preferences over the payment space in Appendix E, followed by the definition, analysis and simulation results of a class of DSIC agent-dependent mechanism termed contingent menu mechanisms (CMM).

A Proofs

A.1 Proof of Lemma 1: Properties of the Expected Utility

(i) From integration by parts:

$$\begin{aligned}
 u(z) &= \mathbb{E}[V^+] + \mathbb{E}[V \cdot \mathbf{1}\{-z \leq V \leq 0\}] - z\mathbb{P}[V < -z] \\
 &= \mathbb{E}[V^+] + \int_{-z}^0 v dF(v) - zF(-z) \\
 &= \mathbb{E}[V^+] + vF(v) \Big|_{-z}^0 - \int_{-z}^0 F(v) dv - zF(-z) \\
 &= \mathbb{E}[V^+] - \int_0^z F(-v) dv
 \end{aligned}$$

(ii) It is easy to know $u(0) = \mathbb{E}[V^+]$ from (4). For the 2nd part, denote $V^- \triangleq \min\{V, 0\}$. We know $V = V^+ + V^-$ thus from the linearity of expectation, $\mathbb{E}[V] = \mathbb{E}[V^+] + \mathbb{E}[V^-]$. To prove $\lim_{z \rightarrow +\infty} u(z) = \mathbb{E}[V]$, we only need to prove

$$\lim_{z \rightarrow +\infty} - \int_0^z F(-v) dv = \mathbb{E}[V^-]. \quad (6)$$

First, observe that the CDF of V^- coincides with that of V for $v < 0$, i.e. $\mathbb{P}[V^- \leq v] = \mathbb{P}[V \leq v]$ for all $v < 0$. Now let $Y = -V^-$, we know the CDF of Y is of the form $G(v) = \mathbb{P}[Y \leq v] = \mathbb{P}[-V^- \leq v] = \mathbb{P}[V^- \geq -v] = 1 - F(-v)$. Since Y is a non-negative random variable, we know that it's expectation can be written as

$$\mathbb{E}[Y] = \int_0^\infty 1 - G(v) dv = \lim_{z \rightarrow +\infty} \int_0^z F(-v) dv$$

therefore (6) holds. This completes the proof of part (ii).

(iii) We know from (4) that the first part of $u(z)$ is a constant and the 2nd part is an integration of a bounded function ($|F(v)| \leq 1$). Thus $u(z)$ is continuous. Taking the derivative of $u(z)$ w.r.t. z :

$$\frac{d}{dz} u(z) = - \frac{d}{dz} \int_0^z F(-v) dv = -F(-z) < 0$$

Thus $u(z)$ is monotonically decreasing. Moreover, $\frac{d}{dz} u(z)$ is monotonically increases in z since $F(-z)$ is decreasing in z , thus $u(z)$ is (weakly) convex.

□

A.2 Proof of Theorem 2

Denote $ut(z) = \mathbb{P}[V \geq -z]$, namely the utilization of this agent at the contingent payment z . We know from Lemma 1 that

$$\frac{d}{dz}u(z) = -F(-z) = -\mathbb{P}[V \leq -z] = ut(z) - 1$$

As discussed in the main text of the paper, we only need to consider the case where SP and CSP allocate the resource to different agents. Suppose agent 1 wins SP and agent 2 wins CSP, we know that $b_{SP,1}^* \geq b_{SP,2}^*$ and $b_{SP,1}^* \leq b_{SP,2}^*$. Therefore

$$\begin{aligned} u_1(0) &= b_{SP,1}^* \geq u_2(0) = b_{SP,2}^* \\ u_1(z_1^0) &= 0 \leq u_2(z_2^0) \leq u_2(z_1^0) \end{aligned}$$

Thus we know (see Figure 3 in the main text)

$$\begin{aligned} u_1(z_1^0) - u_1(0) &\leq u_2(z_1^0) - u_2(0) \\ \Rightarrow \int_0^{z_1^0} (ut_1(v) - 1)dv &\leq \int_0^{z_1^0} (ut_2(v) - 1)dv \\ \Rightarrow \int_0^{z_1^0} ut_1(v)dv &\leq \int_0^{z_1^0} ut_2(v)dv \end{aligned}$$

Since $ut_1(z)$ and $ut_2(z)$ are both monotonically increasing functions of z (form the monotonicity of CDF's,) we must have:

$$z_1^0 \cdot ut_1(0) \leq \int_0^{z_1^0} ut_1(v)dv \leq \int_0^{z_1^0} ut_2(v)dv \leq z_1^0 \cdot ut_2(z_1^0)$$

Under CSP the 2nd highest bid, which is the contingent payment that agent 2 faces must be $z_2 \in [z_1^0, z_2^0]$, we again know from the monotonicity of ut that:

$$ut_{SP,1} = ut_1(0) \leq ut_2(z_1^0) \leq ut_2(z_2) = ut_{SP,2}$$

This completes the proof that CSP increases utilization of the resource, comparing with SP. □

A.3 Dominant Strategy under the γ -CSP Mechanism

Theorem 7 (Dominant Strategy under γ -CSP). *Under the γ -CSP mechanisms, it is a dominant strategy for an agent to bid the summation of the coordinates of the crossing point between her zero-profit curve and $y = \gamma/(1 - \gamma)z$.*

Proof. Fix agent i . Denote the crossing point between agent i 's zero-profit curve and $y = \gamma/(1 - \gamma)z$ as point A , as illustrated in Figure 5a, and denote $b_i^* = z_A + y_A$. Since A is the crossing point of $y = \gamma/(1 - \gamma)z$ and agent i 's zero profit curve, we know that $u_i((1 - \gamma)b_i^*, \gamma b_i^*) = 0$.

We would like to prove that it is a dominant strategy for agent i to bid b_i as defined above in the γ -CSP mechanism. Assume the highest bids from the rest of the agents is $b_{1'}$. If agent i bids b_i^* , and $b_i^* \geq b_{1'}$, agent i is allocated the resource, and gets utility $u_i((1 - \gamma)b_{1'}, \gamma b_{1'}) \geq$

$u_i((1 - \gamma)b_i^*, \gamma b_i^*) = 0$. The agent's utility does not change if she bids anything weakly above $b_{1'}$, and drops to zero if she bids $b_i < b_{1'}$, thus there is no useful deviation.

In the scenario where $b_i^* < b_{1'}$, it is easy to see that increasing her bids to weakly below $b_{1'}$ does not change her utility (which is zero), and bidding higher than $b_{1'}$ the agent gets negative expected utility. This shows that no useful deviation exists, and completes the proof of the Theorem. \square

A.4 Proof of Lemma 3: (P1)-(P5) Characterization

We prove the five parts of the Lemma in a different order from the statement in the main text of the paper.

Part (i) This is an easy implication of IR: if the assigned agent is charged a payment (z^*, y^*) above her zero-profit curve, her expected utility would be negative.

Before proceeding, we state the following lemma, which is useful for proving parts (ii) and (iii).

Lemma 4. Under any mechanism satisfying (P1)-(P5), if there are two agents with identical types, it cannot be the case that given the outcome, both agents have strictly positive utilities.

The proof of the lemma is provided after the proofs of all five parts of Lemma 3 are presented. Now we are ready to prove parts (ii) and (iii).

Parts (ii) and (iii) Assume agent i^* is the assigned agent and her payment determined by the mechanism is (z^*, y^*) . We would like to prove that (z^*, y^*) is weakly above the frontier of the rest of the agents $u_{N \setminus \{i^*\}}(z)$. This immediately implies part (iii). Combined with (i) which implies $u_{i^*}(z^*, y^*) > 0$, we know that agent i^* is on the frontier at z^* , i.e. $\bar{u}_N(z^*) = u_{i^*}(z)$, which gives us part (ii) of the lemma.

Without loss of generality, we name the assigned agent the agent 1. Assume towards contradiction that there exists another agent, say agent 2, s.t. $u_2(z^*, y^*) = a > 0$. Let us call this economy the economy E . Now let us construct another economy which we call E' , where $u'_{-1} = u_{-1}$ in the original economy, but $u'_1 = u_2$. We claim that in economy E' , both agents 1' and 2' must get utility at least a , since otherwise, the agent who gets utility lower than a can report u_1 as her type, gets allocated and charged (z^*, y^*) (this is from anonymity) and improve her utility to a .

However, Lemma 4 tells us that this cannot be the case. Contradiction. This completes the proof of parts (ii) and (iii) of the lemma.

Part (iv) Assume otherwise, that an allocated agent i^* is charged a two-part payment (z^*, y^*) where $y^* < 0$. First, if $z^* \leq 0$ also hold, the expected revenue the mechanism makes $rev(z^*, y^*) = y^* + z^* \mathbb{P}[V_{i^*} < -z^*] < 0$ must hold, thus no-deficit (ND) is violated. What is left to consider is the case where $y^* < 0$ and $z^* > 0$.

Consider now the same economy where the assigned agent i^* is replaced with an agent with type u'_{i^*} , which follows the (w_i, p_i) type model (see Example 1) with $p'_{i^*} = \max\{1 + y^*/(2z^*), 1/2\}$ and w'_{i^*} is any positive real number. Since $y^* < 0$ and $z^* > 0$, we know that $p'_{i^*} \in [1/2, 1)$ is a valid parameter for an (w_i, p_i) type distribution. If the agent is assigned and charged payment (z^*, y^*) , she uses the resource whenever she is able to and her expected utility is

$$u'_{i^*}(z^*, y^*) = w'_{i^*} p'_{i^*} - (1 - p'_{i^*}) z^* - y^* \geq w'_{i^*} p'_{i^*} - (1 - (1 + y^*/2z^*)) z^* - y^* = w'_{i^*} p'_{i^*} - y^*/2z^* > 0.$$

Therefore, in the economy (u'_{i^*}, u_{-i^*}) , if agent i^* is not allocated, or if she is allocated and get utility below $w'_{i^*}p'_{i^*} - y^*/2z^*$, she has an incentive to report u_{i^*} of the original allocated agent as her type, get assigned the resource, and get a higher expected utility.

Denote $rev_{i^*}(z^*, y^*)$ as the expected revenue of the mechanism (i.e. the expected payment made from the allocated agent to the mechanism) when agent i^* is assigned and faces payments (z^*, y^*) :

$$rev_{i^*}(z^*, y^*) = y^* + z^* \mathbb{P}[V_{i^*} < z^*]$$

We know that an agent's expected utility is equal to her utility from using the resource minus the expected revenue. Therefore:

$$u'_{i^*} = w'_{i^*}p'_{i^*} - rev_{i^*}(z^*, y^*) > w'_{i^*}p'_{i^*} - y^*/2z^* \Rightarrow rev_{i^*}(z^*, y^*) < y^*/2z^* < 0.$$

This means that the mechanism runs into deficit in expectation, which contradicts ND. Thus $y^* < 0$ cannot hold, and we conclude part (iv) holds.

Part (v) We know from parts (i) through (iv) that agent i^* resides on the frontier at z^* . When $\bar{u}_{N \setminus \{i^*\}}(z^*) < u_{i^*}(z^*)$, from the convexity of u_{i^*} and the definition of \bar{u}_N , we know that $u_{i^*}(z) = \bar{u}_N(z)$ for z in a small neighborhood around z^* . Therefore, from Lemma 2, the utilization corresponds to the slope of the frontier at z^* :

$$\mathbb{P}[V_{i^*} \geq -z^*] = \left. \frac{d}{dz} u_{i^*}(z) \right|_{z=z^*} + 1 = \left. \frac{d}{dz} \bar{u}_N(z) \right|_{z=z^*} + 1.$$

When there is a tie, i.e. when $\bar{u}_{N \setminus \{i^*\}}(z^*) = u_{i^*}(z^*)$, the frontier might not be differentiable at point z^* , since it is possible that two zero-profit curves cross at z^* and the left derivative and right derivative are not the same. However, from the definition and convexity of $\bar{u}_N(z)$, we know that the left and right derivatives of $\bar{u}_N(z)$ at z^* exists. Moreover, $\left. \frac{d}{dz} \bar{u}_N(z) \right|_{z=z^*-} \leq \left. \frac{d}{dz} \bar{u}_N(z) \right|_{z=z^*+}$, and since $u_{i^*}(z) \leq \bar{u}_N(z)$ for all z , $\left. \frac{d}{dz} u_{i^*}(z) \right|_{z=z^*} \in \left[\left. \frac{d}{dz} \bar{u}_N(z) \right|_{z=z^*-}, \left. \frac{d}{dz} \bar{u}_N(z) \right|_{z=z^*+} \right]$. This shows that the utilization still resides in the range of the derivatives of the frontier of the economy.

With the correspondence of the utilization and the slope of the frontier, and the convexity of the frontier, we know that the larger z^* is, the higher the utilization. We know from parts (i)-(iv) that the highest possible utilization is achieved by charging the highest possible penalty under (P1)-(P5): $z^* \leq z_N^0$ (point A in Figure 7.) This completes the proof of Lemma 3. \square

A.4.1 Proof of Lemma 4

Before proving Lemma 4, we provide two more lemmas that are useful for constructing agent types and proving violation of ND.

Lemma 5 (Payment Range). Under any mechanism that satisfies (P1)-(P5), the two-part payment (z^*, y^*) facing the assigned agent must satisfy

$$y^* \geq \max\{0, -z^*\}. \quad (7)$$

Proof. We have just proved in part (ii) that the base payment y^* facing the assigned agent must be non-negative, thus we only need to prove $y^* \geq -z^*$ must hold. When $z^* \geq 0$, this is also implied by part (ii). When $z^* < 0$ and $y^* < -z^*$, similar to the proof of part (iv), we can construct an agent with (w_i, p_i) model type s.t. $p'_{i^*} = 1 + (y^* - z^*)/2z^*$ and any $w'_{i^*} > 0$ s.t. (z^*, y^*) is in her

IR range but out of her ND range: Replacing the original winner with the agent with type u'_{i^*} as described above, we know that her expected utility facing payment (z^*, y^*) is

$$u'_{i^*} = w'_{i^*} p'_{i^*} - (1 - p'_{i^*}) z^* - y^* = w'_{i^*} p'_{i^*} + (y^* - z^*)/2 - y^* = w'_{i^*} p'_{i^*} + -(y^* + z^*)/2 > w'_{i^*} p'_{i^*}.$$

With similar arguments as in the proof of part (iv), we know that in order for agent i^* with type u'_{i^*} to be willing to report truthfully, u'_{i^*} must be assigned and the mechanism would run into deficit. \square

The following lemma proves that given the expected utility function $u(z)$ of an agent, we can vertically shift the utility function downwards with some adjustment, and still obtain a valid type in the original type space, be it all types satisfying (A1)-(A3), or the set of all (w_i, p_i) types.

Lemma 6. Given the expected utility function $u(z)$ of any type F that satisfies (A1)-(A3), for any constant $0 \leq c < u(0)$, the following function:

$$u_c(z) = \max\{u(z) - c, -z\}$$

is also the expected utility of a valid type F' that satisfy (A1)-(A3). In particular, if F follows the (w_i, p_i) type model, F' is also a type under the (w_i, p_i) type model.

Proof. We prove this lemma by construction. For a general type F , the corresponding expected utility function $u(z)$ and any $c \in [0, u(0)]$, define the threshold T_c as $T_c \triangleq -\inf\{z \in \mathbb{R} : u(z) - c \geq -z\}$. Since $u(0) - c > 0$, we know that $u(0) - c > -0$ holds thus $T_c > 0$. Now define the random variable V_c as the original random variable truncated at T_c , i.e. $V_c = V$ whenever $V \leq T_c$, and $V_c = T_c$ if $V > T_c$. We know from the continuity of $u_c(z)$ that $u_c(z) = -z$ for $z \leq T_c$ and $u_c(z) = u(z) - c$ for $z > T_c$.

Observing $V_c \leq V$ and $T_c > 0$, we know that the distribution of V_c satisfies (A2)-(A3). What is left to prove is (A1) and that the corresponding expected utility function, which we call $u'_c(z)$, is exactly $u_c(z)$. First, since $V_c \leq T_c$, for any penalty level $z < -T_c$ (i.e. when the payment to the agent for no-show is high enough), the agent never uses the resource and always gets paid $-z$ for no-show, thus the agent's expected utility coincides with $u'_c(z) = -z = u_c(z)$ for $z < -T_c$. We know from the continuity of the expected utility function that $u'_c(z) = u_c(z) = -z$ for $z = -T_c$ as well, and $u(-T_c) - u_c(-T_c) = c$ holds.

What is left to prove is $u'_c(z) = u_c(z)$ for $z > -T_c$, since $u'_c(z) = u_c(z)$ implies $u'_c(0) = u(0) - c > 0$ thus (A1) holds. Observing that the CDF of V and V_c coincides at values lower than T_c , i.e. $F(v) = F_c(v)$ for $v < T_c$ we know from Lemma 2 that the derivatives $\frac{d}{dz} u(z) = \frac{d}{dz} u'_c(z)$ for all $z > T_c$, thus $u(z) - u'_c(z) = u(-T_c) - u'_c(-T_c) = c \Rightarrow u'_c(z) = u(z) - c$ for all $z > -T_c$. This completes the proof for the general case.

For the (w_i, p_i) case, assume that the original random variable V takes value $w > 0$ w.p. p and value $-\infty$ w.p. $1 - p$. It is easy to show that the random variable V_c which takes value $w - c/p_i > 0$ (since $c < u(0) = w \cdot p$) w.p. p and $-\infty$ w.p. $1 - p$ has the exact utility function $u_c(z) = \max\{u(z) - c, -z\}$. \square

We are now ready to prove our lemma.

Proof of Lemma 4. Assume towards a contradiction, that in an economy, there exists two agents, 1 and 2, with type $u_1 = u_2$, such that under the outcome of an mechanism satisfying (P1)-(P5), both agents get strictly positive expected utilities. Since we require the mechanisms to be anonymous, we must have $util_1 = util_2 = a > 0$.

First, we prove that ND is violated if $a > u_1(0)/2$. Recall that an agent's expected utility is equal to her utility from using the resource minus her expected payment to the mechanism. For an agent with type u_1 , the highest expected utility she can get from using the resource is to use the resource iff $V_1 \geq 0$, which gives her expected utility $\mathbb{E}[V_1^+] = u_1(0)$. Though the resource can be randomly assigned to either of the two agent, the resource can be consumed by at most one of the agents at a time, thus the expected utility from the two agents combined from using the resource cannot exceed $u_1(0)$. Now if $u_1(0) < 2a$, denote the revenue from agent 1 and 2 as rev_1 and rev_2 respectively, we know that $util_1 + util_2 \leq u_1(0) - rev_1 - rev_2 \Rightarrow rev_1 + rev_2 \leq u_1(0) - util_1 - util_2 < 0$, i.e. the mechanism gets negative expected revenue.

We now claim that if both agents u_1 and u_2 gets expected utility at least a from the outcome of the mechanism and that $a < u_1(0)/2$, then they can also both get expected utility at least $\frac{7}{4}a$. Applying the claim repeatedly, we know that the expected utility agents need to receive would exceed $u_1(0)$, thus ND must be violated, as we proved in the above discussion.

We now prove the claim. First, when both agents are assigned with probability $1/2$, in order for them to get expected utility a , the payment they face, once assigned, must reside below the curve $u_1(z) - 2a$. Now we construct a type $u'_1(z) = \max\{-z, u_1(z) - \frac{3}{2}a\}$. Since $\frac{3}{2}a < u_1(0)$, we know from Lemma 6 that $u'_1(z)$ is a valid type from the same class of types as $u_1(z)$. In the economy where agent 1 is replaced by agent 1', i.e. $E' = (u'_1, u_2, u_{-1,-2})$, we know that if agent 1' reports u_1 as her type, she will get expected utility at least $1/4a$, since she will get allocated with probability $1/2$, and that her payment would be weakly below $u'_1(z) - a/2$ by construction.

Since $u'_1 \neq u_2$, in economy E' , in order for agent 1' to get utility at least $a/4$, agent 1' must be allocated with probability one (from the requirement that the mechanism be deterministic). Denote the payment she face as (z', y') . We know from Lemma 5 that $u'_1(z') = u_1(z') - \frac{3}{2}a$ must be true. Otherwise, $u'_1(z') = -z'$ must hold, and $u'_1(z', y') \geq 1/4a \Rightarrow -z' - y' \geq 1/4a \Rightarrow y < -z'$, thus the mechanism runs into deficit.

Now we examine agent 1 in the original economy $E = (u_1, u_2, u_{-1,-2})$: if agent 1 report u'_1 as her type, she must also be allocated with probability one, and charged the same payment (z', y') . Since $u_1(z') = u'_1(z') + \frac{3}{2}a$, the expected utility of agent 1 must be $u_1(z', y') = u'_1(z', y') + \frac{3}{2}a \geq \frac{1}{4}a + \frac{3}{2}a = \frac{7}{4}a$. Repeat the same argument for agent 2, we completed the proof of the claim. \square

A.5 Uniqueness and Optimality of CSP

A.5.1 Proof of Theorem 3: CSP is Unique

From Lemma 3 and the assumption (P7), we know that in any economy, the payment (z^*, y^*) facing the allocated agent i^* must satisfy $y^* = 0$. W.l.o.g assume agents are ordered in decreasing-order in their zero-crossings $z_1^0 \geq z_2^0 \geq \dots \geq z_n^0$. Consider for now the case with no tie at the highest zero-crossings: $z_1^0 > z_2^0$. Lemma 3 implies that the penalty must fall within $z^* \in [z_2^0, z_1^0]$ and agent 1 must be allocated in order not to violate IR.

We now argue $z^* = z_2^0$ must hold, and thus the CSP outcome. Assume otherwise and $z^* > z_2^0$, consider the economy in which agent 1 is replaced by agent 1' with zero-crossing $z_{1'}^0 > z_2^0$ but $z_{1'}^0 < z^*$. Agent 1' must be allocated from the above argument and (P6), and that the penalty that she faces z' must be smaller than z^* , otherwise her expected utility $u_{1'}(z', 0) \leq u_{1'}(z^*) < u_{1'}(z_{1'}^0) = 0$ which violates IR. This gives agent 1 in the original economy a useful deviation, which is to report the type of agent 1', getting allocated and charged a smaller penalty.

When there is a tie at the highest zero-crossings, Lemma 3 implies that the only possible

outcomes would be one of the agents with the highest zero-crossings getting allocated and payment must be also be the highest zero-crossing $(z_N^0, 0)$ — the only point weakly below the frontier, above the second frontier and with $y = 0$. This is also the CSP outcome. A direct-revelation version of CSP may break ties in favor of the agent with highest utilization at the zero-crossing, and achieve higher utilization than breaking ties uniformly at random. This does not affect incentives since when tied, both allocated and unallocated agents get expected utility zero.

This completes the proof of the uniqueness of CSP. \square

A.5.2 Proof of Theorem 5: CSP is not Dominated

Assume that there is a mechanism M s.t. the utilization under M is always as good as that of CSP for any economy. We proceed in steps to show that M must be identical to CSP.

We know from Lemma 3 that in any mechanism that satisfy (P1)-(P5), the allocated agent must be on the frontier, thus any mechanism that dominate CSP must also allocate to the agent that gets allocated in CSP, ignoring tie-breaking issues for now. Note that this rules out the use of reserve prices, which might leave the resource unallocated. Consider an economy E and the allocated agent in E under mechanism M , say agent 1. In order for mechanism M to dominate CSP in utilization, agent 1 must face a penalty at least as high as the second highest zero-crossing (let's denote it as z_2^0). If the penalty is indeed z_2^0 , then the outcome coincides with that of CSP, and we are all set.

Now assume that agent 1 is charged a payment (z_1, y_1) where the penalty z_1 is higher than the second highest zero-crossing z_2^0 . We first claim that $\bar{u}_{N \setminus \{1\}}(z_1, y_1) > 0$ must hold, i.e. (z_1, y_1) resides above the frontier of the rest of the economy. This is because if $\bar{u}_{N \setminus \{1\}}(z_1, y_1) \leq 0$ were to hold, the fact that $z_1 > z_2^0$ implies $y_1 < 0$, which violates part (iv) of Lemma 3.

Consider now agent $1'$, whose expected utility dominates $\bar{u}_{N \setminus \{1\}}(z)$, but satisfy $u_{1'}(z_1, y_1) < 0$. Such type is guaranteed to exist since (z_1, y_1) resides above $\bar{u}_{N \setminus \{1\}}(z)$, as we have just proved. We know from part (ii) of Lemma 3 that agent $1'$ is the only agent that can be allocated in economy E' with agents $1', 2, \dots, n$, and she has to be charged another two-part payment $(z_{1'}, y_{1'}) \neq (z_1, y_1)$ since the later violates IR for agent $1'$. It is easy to see that if $(z_{1'}, y_{1'}) = (z_2^0, 0)$ which is the second highest zero-crossing, the mechanism is not DSIC for agent 1 in the original economy since $(z_{1'}, y_{1'})$ is a better payment than (z_1, y_1) as $y_1 \geq y_{1'}$, $z_{1'} < z_1$ thus agent 1 has an incentive to misreport $u_{1'}$ as her type.

If $(z_{1'}, y_{1'})$ is not the second highest zero-crossing $(z_2^0, 0)$, it must reside above the horizontal axis. Since we can choose $u_{1'}$ arbitrarily close to $\bar{u}_{N \setminus \{1\}}(z)$, $(z_{1'}, y_{1'})$ must be weakly below $\bar{u}_{N \setminus \{1\}}(z)$, $(z_{1'}, y_{1'})$. $z_1 \neq z_2^0$ therefore implies $z_{1'} < z_2^0$.

Now, we can construct economy E'' , where agent $1''$ follows the (w_i, p_i) type model, s.t. $z_{1''}^0 < z_2^0$ but $u_{1''}(z_{1'}) > y_{1'}$. This is possible since $y_{1'} \leq \bar{u}_{N \setminus \{1\}}(z_{1'})$ as we just proved, and we can choose $p_{1''}$ small enough s.t. $u_{1''}(z)$ has a very sharp slope. In the economy E'' , agent $1''$ must be allocated under mechanism M , whereas under CSP, agent 2 would be allocated, and we can show that utilization under the later is higher since agent 2 has a shallower slope. \square

A.5.3 Proof of Corollary 1

Payments under the γ -CSP mechanism resides on the $y = \frac{\gamma}{1-\gamma}z$ line, and it is a dominant strategy for an agent to bid the summation of the two coordinates of the point at which $y = \frac{\gamma}{1-\gamma}z$ and her zero-profit curve crosses (which always exists and is unique). It's easy to see that γ -CSP allocates the resource to the agent at the crossing point of $y = \frac{\gamma}{1-\gamma}z$ and the frontier (e.g. agent 2 in the

economy depicted in Figure 7) and charges the allocated agent the crossing point of $y = \frac{\gamma}{1-\gamma}z$ and the second frontier $((z_2, y_2) = (z_D, y_D))$ in the Figure 7 example).

As γ decreases from 1 to 0, the penalty part z of the crossing point of $y = \frac{\gamma}{1-\gamma}z$ and the second frontier increases, thus so is the utilization $\mathbb{P}[V_i^* \geq z] = \frac{d}{dz}\bar{u}_N(z) + 1$ due to the convexity of $\bar{u}_N(z)$. Thus CSP which corresponds to 0-CSP achieves the highest utilization for any economy and in particular dominates SP. \square

A.6 Effect of Reserve Prices

We now show that for the simple exponential model and uniform model (see Appendix C,) with simple type distributions as the following, reserve prices do not help with the utilization of a single resource under the CSP+R mechanism if the number of agents in the economy is one. Reserve prices for a single agent auction is equivalent to a posted penalty.

Theorem 8. *For the economy with $n = 1$ agent and each of the following type distributions:*

- (i) *Exponential, with $V = w - O$ where w is the value and O is an exponentially distributed opportunity cost $O \sim \text{Exp}(\lambda)$. The expected opportunity cost $\lambda^{-1} \sim U[0, L]$ for $L > 0$ and the value for using the resource $w \sim U[0, \lambda^{-1}]$.*
- (ii) *Uniform model, with $V \sim U[-a_1, a_2]$. The support $a = a_1 + a_2$ is uniform $a \sim U[0, 1]$. The upper bound is uniformly $b \sim U[0, a/2]$. Let $a_2 = b$ and $a_1 = a - b$.*

The optimal reserve price for achieving highest utilization under the CSP+R mechanism is $R = 0$, i.e. reserve penalties do not help.

Proof. What we need to prove is that for each type model and type distribution, the expected utilization gain by imposing a reserve penalty r when the agent still accepts the resource is smaller than the expected utilization loss when r is larger than the agent's zero-crossing point and the resource is not allocated. We prove this for both models. The exact derivation of an agent's expected utility and the utilization for each model that we use here are provided in Appendix B.

The Exponential Model Denote $\eta = \lambda^{-1}$. For the combination of agent's value w , expected opportunity cost η and the reserve price r s.t. the item is not allocated, the loss of utilization comparing with no reserve price (i.e. giving the resource to the agent for free) is $1 - e^{-\lambda(w+0)}$. When the resource is allocated even when the reserve price is set to r , the improvement in utilization is: $1 - e^{-\lambda(w+r)} - (1 - e^{-\lambda(w+0)}) = e^{-\lambda w} - e^{-\lambda(w+r)}$.

Therefore, on average, the expected gain of utilization when the reserve price is set to r would be

$$\begin{aligned}
G(r) &= -\mathbb{E} \left[\mathbb{1}\{b_{csp}(\eta, w) \leq r\} (1 - e^{-\lambda w}) \right] + \mathbb{E} \left[\mathbb{1}\{b_{csp}(\eta, w) \geq r\} (e^{-\lambda w} - e^{-\lambda(w+r)}) \right] \\
&= \int_0^L \left[\int_w^\eta \left(-\mathbb{1}\{b_{csp}(\eta, w) \leq r\} (1 - e^{-\lambda w}) + \mathbb{1}\{b_{csp}(\eta, w) \geq r\} (e^{-\lambda w} - e^{-\lambda(w+r)}) \right) \frac{1}{\eta} dw \right] \frac{1}{L} d\eta \\
&= L^{-1} \int_0^L \frac{1}{\eta} \left[- \int_0^{\eta+\eta W(-e^{-1-r/\eta})} (1 - e^{-\lambda w}) dw + \int_{\eta+\eta W(-e^{-1-r/\eta})}^\eta (e^{-\lambda w} - e^{-\lambda(w+r)}) dw \right] d\eta \\
&= L^{-1} \int_0^L (-1/e + e^{-1-r/\eta}) d\eta = \frac{1}{e} [-1 + e^{-rL} - rL \cdot \Gamma(0, rL)],
\end{aligned}$$

where $\Gamma(a, z)$ is the upper incomplete Gamma function: $\Gamma(a, z) = \int_z^\infty t^{a-1} e^{-t} dt$. It's easy to see that $G(0) = 0$, which corresponds to the case of having no reserve price. We can also see that the derivative of $G(r)$ w.r.t r

$$G'(r) = -\frac{1}{e} L \cdot \Gamma(0, rL) < 0,$$

since $\Gamma(0, r\Lambda^{-1}) > 0$ for all r . This shows that $G(r) < 0$ for all $r > 0$, thus reserve prices would not be helpful for the exponential type either.

The Uniform Model We know from Appendix B that the zero-crossing, which is the CSP bid for this an agent with uniform model as described above would be:

$$b_{csp}(a, b) = a - b - \sqrt{(a - b)^2 - b^2} = a - b - \sqrt{a^2 - 2ab} \leq a/2$$

therefore for $a < 2r$, we lose all utilization, which is $1/4$ in expectation. For $a \geq 2r$ and for a certain b , the loss of utilization if the resource is allocated would be b/a . The gain from reserve price r if the resource is still allocated would be r/a . The gain in utilization while reserve is r is therefore:

$$\begin{aligned} G(r) &= -\frac{2r}{1} \frac{1}{4} + \int_{2r}^1 \int_0^{a/2} \left[-\mathbb{1}\{b_{csp}(a, b) < r\} \frac{b}{a} + \mathbb{1}\{b_{csp}(a, b) \geq r\} \frac{r}{a} \right] \frac{2}{a} db da \\ &= -\frac{r}{2} + \int_0^1 \frac{2}{a} \left[-\int_0^{\sqrt{2ar}-r} \frac{b}{a} db + \int_{\sqrt{2ar}-r}^{a/2} \frac{r}{a} db \right] da \\ &= r(-r + \log(2) + \log(r)) \end{aligned}$$

The derivatives of $G(r)$ w.r.t. r is therefore:

$$G'(r) = 1 - 2r + \log(2) + \log(r) \text{ and } G''(r) = -2 + \frac{1}{r} \geq 0$$

for all $0 < r \leq 1/2$ thus $G(r)$ is convex. It's also easy to check that $G(0) = 0$ and $G(1/2) = -0.25$, thus $G(r) \leq 0$ always hold which means that the reserve price is never helpful. \square

A.7 CSP+R Dominates SP+C

We first provide the formal definition of the SP+C mechanism.

Definition 8 (Second-price with fixed penalty C (SP+C)). The *SP+C mechanism* collects a single bid $b_i \geq 0$ from each agent. Let $b = (b_1, \dots, b_n)$ be a bid profile.

- Allocation rule: $x_{i^*}(b) = 1$ for $i^* \in \arg \max_{i \in N} b_i$, breaking ties at random.
- Payment rule: For i^* : $t_{i^*}(b) = \max_{i \neq i^*} b_i$, $t_{i^*}^{(1)}(b) = 0$, $t_{i^*}^{(0)} = C$ and for $i \neq i^*$, $t_i(b) = 0$.

We know that under the SP+C mechanisms, at period one, the allocated agent will choose to use the resource iff the realized value $v_{i^*} \geq -C$, thus the expected utility for each agent from the right for using the resource is $\mathbb{E}[V_i \cdot \mathbb{1}\{V_i \geq -C\}] - C \cdot \mathbb{P}[V_i < -C]$, and it is a dominant strategy for agent i to bid

$$b_{i,SP+C}^* = \mathbb{E}[V_i \cdot \mathbb{1}\{V_i \geq -C\}] - C \cdot \mathbb{P}[V_i < -C], \quad (8)$$

if this value is non-negative, and zero otherwise (equivalent to not participating.) The *second price auction* (SP) is attained as a special case, by setting the penalty $C = 0$.

Now we are ready to prove that the utilization under SP+C mechanism is always weakly lower than that of the CSP+R mechanism, profile by profile, if R and C are set to be equal.

Theorem 9 (CSP+R Dominates SP+C). *For any set of agent types satisfying (A1)-(A3), the utilization of CSP+R is higher than that of SP+C in dominant strategy, if R is set to be equal to C .*

Proof. Since we set the reserve penalty of CSP as C , let's call this mechanism as CSP+C. First, observe that as long as the reserve penalty is met, CSP+C always allocates the resource to the same agent that gets allocated under CSP. Also observe that under SP+C, the dominant strategy is to bid $b_{i,SP+C}^* = \max u_i(C), 0$ (see (8).) W.l.o.g., assume $z_1^0 \geq z_2^0 \geq \dots \geq z_n^0$. We consider the following cases:

- (i) $C > z_1^0$: the resource is not allocated in the CSP+C mechanism since the reserve penalty is not met. In this case, $u_i(C) < 0$ for all i thus no agent submits non-negative bid under the SP+C mechanism. Therefore, zero utilization is achieved in both cases.
- (ii) $z_2^0 \leq C \leq z_1^0$: both the SP+C and CSP+R mechanism allocate the resource to agent 1, and the penalty payment are both set to be C , thus the two mechanisms achieve the same utilization.
- (iii) $C < z_2^0$: we need to consider whether the two mechanisms allocate the resource to the same agent.
 - CSP+C and SP+C allocate the resource to the same agent. If this is the case, the allocated agent has to be agent 1. In SP+C agent 1 is charged penalty C , whereas in CSP+C agent 1 is charged penalty $z_2^0 > C$, therefore the utilization under CSP+C is weakly higher since higher penalty induces higher probability of utilizing the resource for the same agent.
 - SP+C allocates the resource to a different agent (say agent i_{SP}^*) from that of CSP, agent 1. We must have $u_1(C) \leq u_{i_{SP}^*}(C)$ and $z_1^0 \geq z_{i_{SP}^*}^0$. With the same argument on the convexity of the utility functions as in the proof of Theorem 2, we can show that SP+C has lower utilization than CSP+C.

□

B Bids and Bid Distributions under Different Type Models

B.1 The (w_i, p_i) Model

The expected utility of agents whose type follow the (w_i, p_i) model is of the form:

$$u(z, y) = \begin{cases} w_i p_i - (1 - p_i)z - y, & \text{for } z \geq -w_i \\ -z - y, & \text{for } z < -w_i \end{cases},$$

since considering negative z , the agent uses the resource only if $w_i \geq -z$, and decides to never use the resource, gets paid z and pay y if $w_i < -z$. The zero-crossings, which correspond to SP and CSP bids are:

- Crossing with the vertical axis: $b_{SP} = y^0 = -w_i p_i$
- Crossing with the horizontal axis: $b_{CSP} = z^0 = w_i p_i / (1 - p_i)$

Bids under other mechanisms that we discussed are:

- SP+C: $b_{SP+C} = u(C, 0) = w_i p_i - (1 - p_i)C$,
- γ -CSP: $b_{\gamma CSP} = (1 - \gamma)w_i p_i / (1 - p_i + \gamma p_i)$,

The zero-profit curve is given by $y = w_i p_i - (1 - p_i)z$, and the zero-revenue curve is given by $y = -(1 - p_i)z$, as illustrated in Figure 5b(i). Since the IR curve and ND curve never cross, the maximal penalty z we can charge an agent with (w_i, p_i) type s.t. the mechanism is IR and ND BB would be ∞ . The utilization, however, would always be $ut(z, y) = p_i$, as long as $z \geq -w_i$.

B.2 The Uniform Model

Assume that the random value for an agent is uniformly distributed $V \sim U[-a_1, a_2]$ with parameters a_1, a_2 s.t. $-a_1 < 0 < a_2$ and $a_1 > |a_2|$. The CDF and PDF of V are given by

$$F(v) = \begin{cases} 0, & \text{for } v < -a_1 \\ \frac{v - a_1}{a_2 - a_1}, & \text{for } -a_1 \leq v < a_2 \\ 1, & \text{for } v \geq a_2 \end{cases}, f(v) = \begin{cases} \frac{1}{a_1 + a_2}, & \text{for } -a_1 \leq v \leq a_2 \\ 0, & \text{otherwise} \end{cases}$$

and the agent's expected utility as a function of the two-part payment is:

$$u(z, y) = \begin{cases} -z - y, & \text{for } z \leq -a_2 \\ \frac{z^2 - 2a_1 z + a_2^2}{2(a_1 + a_2)} - y, & \text{for } -a_2 < z \leq a_1 \\ (a_2 - a_1)/2 - y, & \text{for } z > a_1 \end{cases}$$

Zero-Crossings, which corresponds to the SP and CSP bids are:

- Zero-crossing with the vertical axis: $b_{SP} = y^0 = u(0) = a_2^*/2(a_1 + a_2)$.
- Zero-crossing with the horizontal axis: $b_{CSP} = z^0 = a_1 - \sqrt{a_1^2 - a_2^2}$.

and the bids under other mechanisms that we discussed are:

- SP+C: $b_{SP+C} = u(C, 0)$.
- γ -CSP: $b_{\gamma CSP} = (a_1 + \gamma a_2 - \sqrt{a_1^2 - a_2^2} + 2\gamma a_1 a_2 + 2\gamma a_2^2) / (1 - \gamma)$.

Facing two-part payment (z, y) , utilization is of the form:

$$ut(z) = \begin{cases} 0, & \text{for } v < -a_2 \\ (z + a_2)/(a_1 + a_2), & \text{for } -a_2 \leq v < a_1 \\ 1, & v \geq a_1 \end{cases}$$

and the expected revenue of the mechanism is:

$$rev(z, y) = z \cdot \frac{-z + a_1}{a_1 + a_2} + y$$

First best price, i.e. the two-part payment with the highest penalty component in the intersection of IR and ND ranges are: $z^{FB} = a_2$, $y^{FB} = 2a_2(a_2 - a_1)$. The utilization achieved under the FB payment would be

$$ut_{FB} = 2a_2 / (a_1 + a_2).$$

B.3 The Exponential Model

Consider the model where the value for using the resource equals to a fixed positive number w minus an exponentially distributed opportunity cost O with parameter λ :

$$V = w - O \text{ where } O \sim \text{Exp}(\lambda)$$

The CDF and PDF of the random value is given by:

$$F(v) = \begin{cases} e^{\lambda(x-w)}, & \text{for } v \leq w \\ 1, & \text{for } v > w \end{cases}, \quad f(v) = \begin{cases} \lambda e^{\lambda(v-w)}, & \text{for } v \leq w \\ 0, & \text{for } v > w \end{cases}$$

The agent's expected utility as a function of the two-part payments are:

$$u(z, y) = \begin{cases} \int_{-z}^w v \cdot \lambda \cdot e^{\lambda(v-w)} dv - z \cdot e^{-\lambda(z+w)} - y = w + \frac{1}{\lambda} (e^{-\lambda(w+z)} - 1) - y & \text{for } z \geq -w \\ z - y, & \text{for } z < -w \end{cases}.$$

The zero-crossings, which correspond to bids under the SP and CSP mechanisms, are:

- Zero-crossing with the vertical axis: $b_{\text{SP}} = y^0 = w + \frac{1}{\lambda} (e^{-\lambda w} - 1)$.
- Zero-crossing with the horizontal axis: $b_{\text{CSP}} = z^0 = -w - \frac{1}{\lambda} \log(1 - w\lambda)$.

These are solved by looking for roots of $u(z, 0) = 0$ and $u(0, y) = 0$. Bids under other mechanisms that we discussed are:

- SP+C: $b_{\text{SP+C}}^* = w + \frac{1}{\lambda} (e^{-\lambda(w+C)} - 1)$.
- γ -CSP:

$$\begin{aligned} b_{\gamma\text{CSP}} &= \frac{1}{\gamma\lambda} \cdot [-1 + \gamma + w\lambda - w\gamma\lambda] + \frac{1}{\lambda} \cdot W\left(-\frac{e^{-1+1/\gamma-w\lambda/\gamma}(-1+\gamma)}{\gamma}\right) \\ &= \frac{(w\lambda - 1)(1 - \gamma)}{\gamma\lambda} + \frac{1}{\lambda} W\left(-\frac{e^{-1+1/\gamma-w\lambda/\gamma}(-1+\gamma)}{\gamma}\right) \end{aligned}$$

where $W(\cdot)$ is the Lambert W function (also called the product log function), which is defined as the inverse of $z = we^w$.

An agent's utilization is of the form:

$$ut(z) = \begin{cases} 0, & \text{for } z < -w \\ 1 - e^{-\lambda(w+z)}, & \text{for } z \geq -w \end{cases},$$

and the expected revenue of the mechanism is:

$$rev(z, y) = z \cdot e^{-\lambda(w+z)} + y$$

Setting $ut(z) = rev(z, 0)$ (i.e. solving for the crossing point of the zero-profit curve and the zero-revenue curve, as illustrated in Figure 5b(ii)), we get the highest possible penalty that a mechanism can charge an agent with exponentially distributed type as:

$$z^{FB} = -\frac{1}{\lambda} - \frac{1}{\lambda} W\left(k, e^{-1+w\lambda}(-1+w\lambda)\right).$$

Here, we need to take $k = -1$, since when $k = 0$, $W(k, e^{-1+w\lambda}(-1+w\lambda)) = -1+w\lambda \Rightarrow z^{FB} = -w$, which is not what we're looking for.

C Additional Simulation Results

We provide here additional simulation results for different type models and distributions of agents’ types, for both single and multi-resource allocation.

C.1 Exponential Model

First, we look at some detailed profile-by-profile comparison for different mechanisms under the same exponential model as we had studied in the main text of the paper.

Recall that an agent’s value for using the resource is $V_i = w_i - O_i$ where $w_i > 0$ is the fixed value and $O_i \sim \text{Exp}(\lambda_i)$ is the opportunity cost. $\mathbb{E}[V_i] = w_i - \lambda_i^{-1}$ where λ_i^{-1} is the expected value of the opportunity cost. We consider the type distribution where the value and the expected opportunity cost λ_i^{-1} are uniformly distributed: $\lambda_i^{-1} \sim U[0, L]$ and $w_i \sim U[0, \lambda_i^{-1}]$. For results in this subsection, we set $L = 10$.

C.1.1 Single Resource Allocation

SP vs CSP Fixing the number of agents to five, we zoom in and compare the utilization for the two mechanisms profile-by-profile, as the scatter plot in Figure 11 (this is a similar plot to Figure 9b). The horizontal coordinate for each point is the utilization under CSP for an economy, and the vertical axis is the utilization for the same economy under SP.

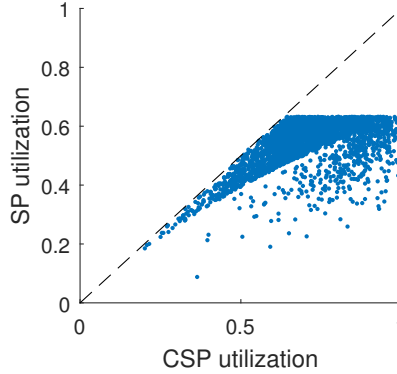


Figure 11: Utilization comparison between SP and CSP for exponential model.

As predicted by Theorem 2, the utilization under CSP is always higher. The dense “triangle” generally correspond to instances when SP allocates the resource to the same agent that gets allocated under CSP, and the rest of the points on the lower right side are instances in which SP and CSP allocate to different agents.

Effect of reserve price in CSP+R To get more intuition on the effect of reserve prices, Figure 12 also compares the utilization of CSP and CSP+R, profile-by-profile, for different reserve prices.

For $R > 0$, then when the resource is allocated under CSP+R the utilization under CSP+R can be higher than under CSP. As R increases, there is a higher improvement in utilization if the resource is allocated under CSP+R, however, there are a lot more profiles in which the resource is not allocated under CSP+R, where the utilization under CSP+R drops to zero (see the data points

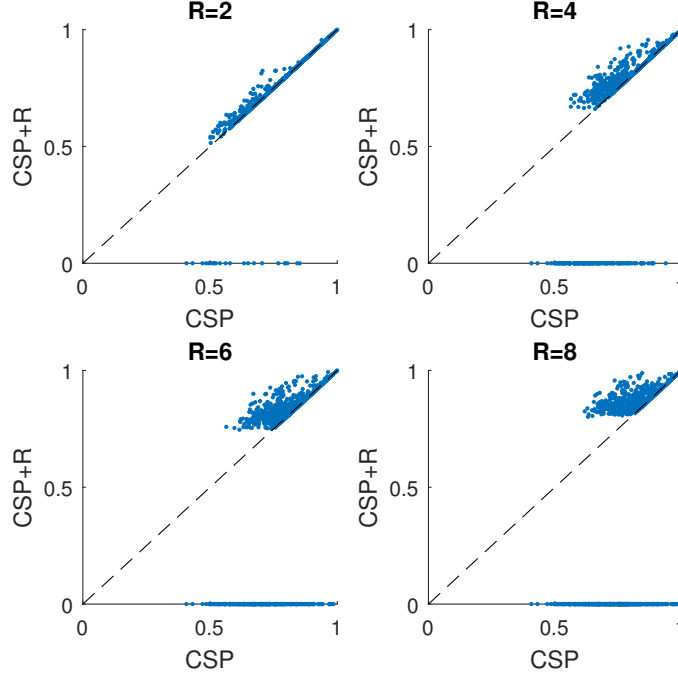


Figure 12: Utilization in CSP vs CSP+R with different reserve prices R for the exponential model.

on the “floor”). The overall effect is that the average utilization under CSP+R is lower, relative to $R = 0$, as we have shown in Figure 9c.

C.1.2 Allocating Multiple Resources

Detailed comparison of GCSP and VCG As we have showed in the main text of the paper, the generalized CSP mechanism (GCSP) does not necessarily have higher utilization than VCG for allocating multiple heterogeneous resources. Figure 9, GCSP achieves higher utilization on average, and the fraction of economies where VCG actually achieves higher utilization is very small.

Under the same experimental setup as in Section 6, we compares utilization of the two mechanisms profile-by-profile in Figure 13, varying the number of agents in the economy. As we expected, utilization under both mechanisms increase as the number of agents in the economy increases. Since competition increases assigned agents’ penalties, utilization under GCSP is significantly boosted and almost always dominates that of VCG with a larger number of agents in the economy.

Allocation of $m = 2$ Resources Under the same setup, but change the number of resources to be allocated to $m = 2$, the average utilization over 1000 randomly generated economies are as plotted in Figure 14a, and the fraction of economies for which GCSP or VCG achieves higher utilization than the other are as plotted in Figure 14b. We can see that the results are consistent with those presented in Section 6.

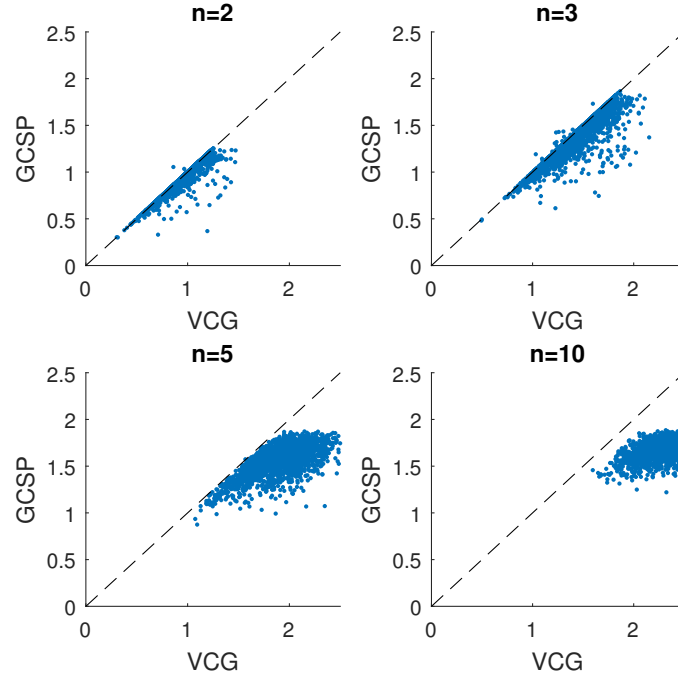


Figure 13: Utilization comparison between the VCG mechanism and the Generalized CGP mechanism for allocating 3 resources, varying the number of agents.



Figure 14: Utilization for assigning two resources under the exponential type model.

C.2 Uniform Model

We present in this subsection utilization for single resource allocation for the uniform model as introduced in Appendix B.2. Each agent's value is assumed to be uniformly distributed on some interval parameterized by $0 < a_2 < a_1$.

$$V_i \sim U[-a_{1,i}, a_{2,i}]$$

Assume that $a_{1,i}$'s are iid uniformly distributed on $[0, 10]$ and that given $a_{1,i}$, $a_{2,i}$ is uniformly distributed on $[0, a_{1,i}]$ (s.t. $\mathbb{E}[V_i] < 0$ and dominant strategies exist).

SP, CSP and Benchmarks The average utilization of 10,000 randomly generated economies under various mechanisms and benchmarks are presented in Figure 15. The First Best, P1-P5 UB and Random are benchmarks computed in the same way as introduced in Section 6.



Figure 15: Utilization for assigning one single resource under the uniform type model.

Similar to the results that we see for exponential distribution, we see that there are gaps between the full information first best, upper bound on what can be achieved under (P1)-(P5), and the average utilization of the CSP mechanism. CSP achieves utilization way higher than the SP and random mechanisms. A detailed profile by profile comparison between SP and CSP when the number of agents is $n = 3$ is provided as Figure 15b.

C.3 (w_i, p_i) Model

The last type model we examine are the (w_i, p_i) model as introduced in Example 1. For the allocation of a single resource, an agent's value V_i for using the resource takes some fixed positive number w_i with probability p_i . With probability $1 - p_i$, V_i takes value $-\infty$, modeling the hard constraint that the agent is unable to use the resource.

We consider the most simple type distribution, agents' values w_i are i.i.d. uniformly distributed on $[0, 1]$: $w_i \sim U[0, 1]$, and where p_i 's are also i.i.d. uniformly distributed on $[0, 1]$: $p_i \sim U[0, 1]$.

CSP, SP and other Benchmarks For the type model and type distribution described as above, varying the number of agents from 2 to 15, the average utilization over 10,000 randomly generated economies under various mechanisms are as shown in Figure 16a. Benchmarks First Best, P1-P5 UB, and Random are defined in the same way as in Section 6.

First, what we see is a significant improvement from both SP and CSP in comparison with allocating uniformly at random. The reason SP achieves good performance as well, especially when



Figure 16: Utilization for assigning two resources under the (w_i, p_i) type model.

the number of agents is large, is that the bids under SP $b_{i,SP}^* = w_i p_i$ is positively correlated with p_i . When the number of agents in the economy is large enough, p_i of the SP winner is also more likely to be large. CSP still achieves an improvement over SP. Moreover, utilization under CSP now coincides with the upper bound under (P1)-(P5), as we have proved in Theorem 4.

A detailed, profile by profile comparison of utilization under SP and CSP when the number of agents is $n = 3$ is presented in Figure 16b. Since penalties does not improve utilization under the (w_i, p_i) model, when SP and CSP allocate to the same agent, the utilization would be the same, thus reside on the 45 degree diagonal. When CSP allocates to a different agent, the utilization would be improved (see the off-diagonal points).

D Welfare Considerations

We briefly discuss in this section the alternative model, where the society gains a fixed amount $C > 0$ when the resource is utilized, and the objective is welfare maximizing. We first prove that the modifying the SP+C mechanism (where C is set to be the societal welfare gain) to allow negative bids, i.e. to allow the mechanism to make positive transfers to the agents, SP+C mechanism is optimal for maximizing social welfare.

Theorem 10 (Optimality of SP+C). *Let $\{F_i\}_{i \in N}$ be any set of bidders' value distributions. When society gains C from the resource being utilized, the SP+C mechanism with negative bids is DSIC, IR and welfare optimal.*

Proof. We first analyze what is the welfare optimal way of using a resource. Fix an agent with random value V and CDF F . If the resource is not allocated, the social welfare is zero. If the resource is allocated to the agent and the agent is charged a no-show penalty z , the agent decides to use the resource iff $V \geq -z$ (breaking ties in favor of using the resource), therefore the utilization is:

$$ut(z) \triangleq \mathbb{P}[V \geq -z]$$

and the expected social welfare, which is equal to what the agent gets from using the resource plus the societal welfare gain, would be:

$$sw(z) = \mathbb{E}[V \cdot \mathbb{1}\{V \geq -z\}] + z \cdot ut(z)$$

From the welfare-maximization's perspective, when an agent uses the resource, the welfare gain is $V + C$. When an agent does not use the resource, the welfare gain is zero. Therefore, to maximize welfare, the agent should use the resource iff $V \geq -C$. This decision can be induced by setting the no-show penalty of this agent as C . Now we know that for an agent with type F_i , the maximal welfare gain from allocating the resource to the agent is

$$\begin{aligned} sw_i^* &= \mathbb{E}[V_i \cdot \mathbb{1}\{V_i \geq -C\}] + C \cdot ut(C) \\ &= \mathbb{E}[V_i \cdot \mathbb{1}\{V_i \geq -C\}] + C \cdot \mathbb{P}[V_i \geq -C] \\ &= \mathbb{E}[V_i \cdot \mathbb{1}\{V_i \geq -C\}] - C \cdot \mathbb{P}[V_i < -C] + C \\ &= b_{i,SP+C}^* + C, \end{aligned}$$

since an agent's DSE bid under the SPC mechanism is $b_{i,SP+C} = \mathbb{E}[V_i \cdot \mathbb{1}\{V_i \geq -C\}] - C \cdot \mathbb{P}[V_i < -C]$. This shows that the when negative bids are allowed (since $b_{i,SP+C}^*$ might not be positive when C is large), the mechanism allocates the resource to the agent with highest sw_i^* . Combining with the fact that the penalty the allocated agent face is indeed C , we complete the proof of this theorem. \square

D.1 Welfare Characterization under (P1)-(P5)

We characterize in this section the set of all achievable social welfare given a mechanism that satisfies (P1)-(P5), and show that when C is very large, the utilization maximizing mechanism is also welfare-maximizing. First, denote the *agent welfare* $aw(z) \triangleq \mathbb{E}[V \cdot \mathbb{1}\{V \geq -z\}]$ as the welfare gain from the agent's perspective, from using the resource when the penalty is z . The social welfare is therefore of the form:

$$sw(z) = aw(z) + C \cdot ut(z).$$

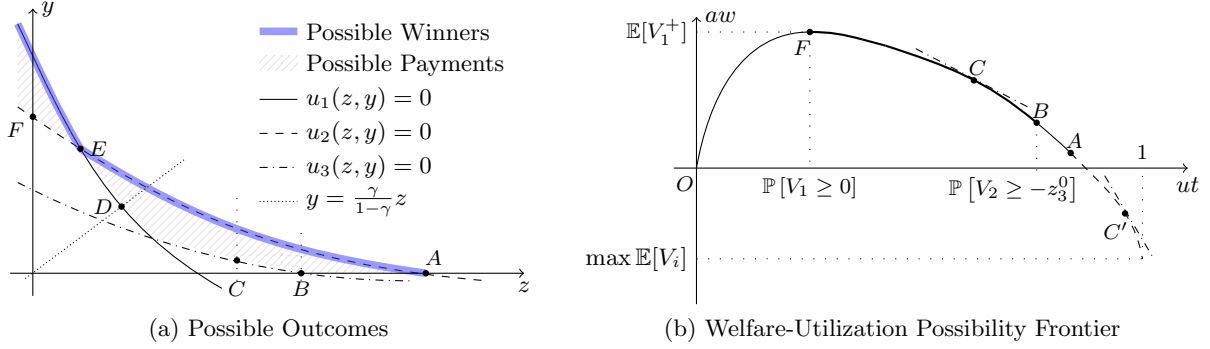


Figure 17: Characterization of possible outcomes and utilization for mechanisms satisfying (P1)-(P5).

Recall from Lemma 3 that given a mechanism where (P1)-(P5) are satisfied, the allocated agent must reside on the frontier of all agents, and that the utilization correspond to the slope of the frontier:

$$ut(z) = \frac{d}{dz} \bar{u}_N(z) + 1.$$

As the penalty component z^* of the payment facing the allocated agent moves from $-\infty$ to $+\infty$, the agent-welfare achieved by the agent on the frontier and the probability that the agent on the frontier is going to use the resource, are as depicted in Figure 17b. Points F , D , C , B in Figure 17b correspond to the payment that the allocated agent faces under the mechanisms SP, γ -CSP, SP+C (with some small C), and CSP respectively, and A is the upper bound under (P1)-(P5).

When the penalty $z = -\infty$, i.e. paying agents an infinite amount for no-show, no agent ever uses the resource thus $aw(-\infty) = ut(-\infty) = 0$. As z increases from $-\infty$ to 0, $aw(z)$ and $ut(z)$ both increases, and at $z = 0$ agent welfare is maximized (point F , achieved by SP). As z increases from 0 to ∞ , $ut(z)$ keeps increasing and $aw(z)$ decreases, until when $z = \infty$, the resource is allocated to the agent with highest $\mathbb{E}[V_i]$ (the agent on the frontier at $z \rightarrow \infty$ and utilization approaches 1).

We can show that at any point on the agent-welfare - utilization frontier, the slope of the frontier $aw \sim ut$ is equal to the penalty z that facing the allocated agent. The Solid line is all possible outcomes under (P1)-(P5), and if we relax the no-deficit (ND) constraint, we are able to achieve the dashed line by paying people in expectation, boosting utilization to 1.

The outcome under SP+C is the point at which $aw \sim ut$ is tangent to the line $aw + C \cdot ut = \text{const}$, which maximizes the social welfare $sw = aw + c \cdot ut$ if the society derives utility C whenever the resource is utilized. When C is large, the tangent point may fall on the dashed segment of the curve (e.g. point C') thus the social welfare maximizing outcome cannot be achievable by any mechanism under (P1)-(P5), since the no-deficit constraint would be violated. This shows that our problem cannot be trivially solved, by designing mechanisms to maximize $sw = aw + C \cdot ut$, and then send $C \rightarrow \infty$.

With the ND constraint, and when C is large, it is easy to see that the welfare-maximizing outcome within the solid segment, i.e. outcomes that are allowed under (P1)-(P5), would be achieved at point A , which is the utilization maximizing point under (P1)-(P5). We also know that the higher utilization is, the higher total welfare is going to be. As we have discussed in the body of the paper, the outcome at point A cannot be implemented in an incentive compatible manner, and the best we can do is B , which is the CSP outcome.

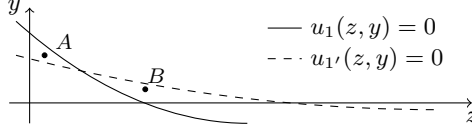


Figure 18: Two possible, zero iso-profit curves for an agent.

E Agent-Dependent Mechanisms

In traditional auction problems where payments are one-dimensional, agent-independence is a necessary condition for DSIC, assuming agents' utilities decrease as the payments increase. Assume otherwise s.t. there exists an economy where agent 1 gets allocated and charged t . Replacing agent 1 with agent 1', 1' also gets allocated and is charged t' . Without loss of generality, assume $t < t'$, agent 1' would have incentive to report as agent 1, get allocated the resource, pay t and get $u_{1'}(t) > u_{1'}(t')$. What is critical here, is that there is an order on the set of all possible payments an agent could be charged, s.t. all agents agree on.

However, agent-independence is not a necessary condition for DSIC because agents have heterogeneous preferences when payments reside in a two-dimensional space, as shown by the example in Figure 18. For type F_1 and iso-profit curve $u_1(z, y) = 0$, agent 1 is allocated and charged two-part payment A . For type F'_1 and iso-profit curve $u_{1'}(z, y) = 0$, agent 1 is allocated and charged two-part payment B . Because $u_1(z_A, y_A) \geq u_1(z_B, y_B)$ and $u_{1'}(z_A, y_A) \leq u_{1'}(z_B, y_B)$, no agent has incentive to pretend to be the other, thus this remains truthful.

As we have discussed in the body of the paper, if the two-part payments are constrained on a domain on which s.t. there exists an order that all agents agree on, agent-independence becomes a necessary condition for DSIC to hold, as in the one dimensional payment space. All mechanisms we have talked about so far restricts payments in some way s.t. agent-independence must hold:

- SP: payments are restricted to $\{(z, y) | z = 0\}$, and the utility of any type monotonically decreases as y increases.
- SP+C: payments are restricted to $\{(z, y) | z = C\}$, and the utility of any type monotonically decreases as y increases.
- CSP: payments are restricted to $\{(z, y) | y = 0\}$, and the utility of any type monotonically decreases as z increases.
- γ -CSP: payments are restricted to $y = \frac{\gamma}{1-\gamma}z$, and the utility of any type monotonically decreases as z (and also y , since y is a monotonically increasing function of z), increases.

E.1 CSP is the Optimal Agent-Independent Mechanism

We first prove that CSP is the optimal agent-independent mechanism under (P1)-(P6), in terms of utilization, profile by profile.

Theorem 11. *For any set of agent types satisfying (A1)-(A3), CSP achieves highest utilization among all mechanisms that satisfy (P1)-(P6) and where the payments are agent-independent.*

Proof. First, Lemma 3 shows that the two-part payment facing any allocated agent must be weakly above the second frontier. We now prove that the payment cannot be strictly above the second frontier.

Assume otherwise, that agent 1 is allocated in some economy E and charged a two-part payment (z_1, y_1) that is strictly above $u_{N-\{1\}}(z)$. Consider the economy E' with agents $1', 2, \dots, n$ where $2, \dots, n$ belong to the original economy E , and where the zero-profit curve of agent $1'$ strictly dominates $u_{N-\{1\}}(z)$, but resides below (z_1, y_1) . This is always possible, observing that $u_{N-\{1\}}(z)$ corresponds to a valid type under (A1)-(A3).

We know from Lemma 3 that agent $1'$ must be allocated in economy E' , since she is the only agent that resides on the frontier, and that the resource must be allocated under (P6). We know from agent-independence that agent $1'$ must be charged (z_1, y_1) as well, thus IR is violated. The proof of the theorem now follows, observing that payments must be on the second frontier and that the CSP payment has the largest penalty component among all such two-part payments. \square

E.2 Contingent Menu Mechanisms

The example in Figure 18 shows the possibility to charge allocated agents different payments that depend on their reported types, as long as the payment is “agent-maximizing” for each allocated agent s.t. there is no incentive to report as another type. Inspired by the critical payment mechanisms for cases when payments are one dimensional, we now introduce a class of mechanisms which we call *contingent menu mechanisms* (CMM), which offers a menu of payments to choose from, or, as a DRM, chooses from a menu of payments for each allocated agent the payment that maximizes the utility for the agent, bases on her own report. The menu of payments that we offer, however, would not depend on the agent’s own report.

We denote the set of payment menus associated with each CMM as Π where each menu $\pi \in \Pi$ is a subset of \mathbb{R}^2 and is not necessarily singleton. First we state a few properties for the price menu set:

- B1. Each menu $\pi \in \Pi$ is defined as a *closed* subset of \mathbb{R}^2 s.t. we can define the utility of an agent with utility function $u(z, y)$ at price menu π as:

$$u(\pi) = \sup_{(z, y) \in \pi} u(z, y)$$

Intuitively, we choose the “agent-maximizing” price out of each menu, according to the reported utility function of the agent.

- B2. (Unique zero-crossing) For an agent with any type and the corresponding utility function $u(z, y)$, there exists a unique menu $\pi \in \Pi$ s.t.

$$u(\pi^*) = \sup_{(z, y) \in \pi^*} u(z, y) = 0 \tag{9}$$

- B3. (Monotonicity) There is a total order, say \succeq defined on the set Π . For any $\pi_1, \pi_2 \in \Pi$ satisfying $\pi_1 \succeq \pi_2$, we must have

$$u(\pi_1) \leq u(\pi_2)$$

for any utility function u induced by any CDF satisfying B1 - B3.

Since we focus on dominant strategy equilibrium, we consider direct-revelation mechanisms w.l.o.g. Denote the set of agents as $N = \{1, 2, \dots, n\}$ and a reported type profile as $\hat{u} = (\hat{u}_1, \dots, \hat{u}_n)$ (recall that utility curves uniquely determine agents’ types), we now formally define the mechanism:

Definition 9 (Contingent Menu Mechanism). A contingent menu mechanism is equipped with a set of menus Π satisfying B1-B3 introduced above, with the following rules:

- *Allocation rule*: allocate the resource to the agent with the “highest” menu according to \succ :

$$x(\hat{u}) = i^* = \arg \max_{i \in N} \pi_i^*$$

where π_i^* is the unique zero-crossing menu of agent i .

- *Payment rule*: charge the allocated agent the payment in the second highest menu s.t. her utility is maximized:

$$t_{i^*}(\hat{u}) = \arg \max_{(z,y) \in \max_{i \neq i^*} \pi_i} \hat{u}_{i^*}(z, y)$$

When menus are singleton sets, the corresponding CMM is actually agent-independent, therefore we are able to interpret some of the mechanisms that we had been talking about as CMM's. See the following example:

Example 8 (Agent-independent CMM's).

- Second Price Auction (SP)
 - The set of payments are singleton sets parameterized by non-negative real numbers: $\Pi = \{\pi_\alpha\}_{\alpha \in \mathbb{R}_{\geq 0}}$ where $\pi_\alpha = \{(0, \alpha)\}$.
 - The total order \succeq is defined as: $\pi_{\alpha_1} \succeq \pi_{\alpha_2}$ iff $\alpha_1 \geq \alpha_2$.
- Second Price with fixed penalty C (SP+C):
 - The set of payments are singleton sets parameterized by real numbers: $\Pi = \{\pi_\alpha\}_{\alpha \in \mathbb{R}}$ where $\pi_\alpha = \{(C, \alpha)\}$.
 - The total order \succeq is defined as: $\pi_{\alpha_1} \succeq \pi_{\alpha_2}$ iff $\alpha_1 \geq \alpha_2$.
- Contingent Second Price (CSP)
 - The set of payments are singleton sets parameterized by non-negative real numbers: $\Pi = \{\pi_\alpha\}_{\alpha \in \mathbb{R}_{\geq 0}}$ where $\pi_\alpha = \{(\alpha, 0)\}$.
 - The total order \succeq is defined as: $\pi_{\alpha_1} \succeq \pi_{\alpha_2}$ iff $\alpha_1 \geq \alpha_2$.
- γ -CSP
 - The set of payments are singleton sets parameterized by non-negative real numbers: $\Pi = \{\pi_\alpha\}_{\alpha \in \mathbb{R}_{\geq 0}}$ where $\pi_\alpha = \{((1-\gamma)\alpha, \gamma\alpha)\}$.
 - The total order \succeq is defined as: $\pi_{\alpha_1} \succeq \pi_{\alpha_2}$ iff $\alpha_1 \geq \alpha_2$.
- Arbitrary payment independence mechanism:
 - The set of payments are singleton sets parameterized by non-negative real numbers: $\Pi = \{\pi_\alpha\}_{\alpha \in \mathbb{R}_{\geq 0}}$ where $\pi_\alpha = \{(z(\alpha), y(\alpha))\}$ and z and y are continuously monotonically increasing functions of α .
 - The total order \succeq is defined as: $\pi_{\alpha_1} \succeq \pi_{\alpha_2}$ iff $\alpha_1 \geq \alpha_2$.

Note that 1) in SP+C, we are allowing agents' zero-crossing menu to be parameterized by negative numbers, i.e. submitting negative bids, which enables the mechanism to be welfare maximizing as discussed in Appendix D and 2) in order to satisfy B1-B3, $z(\alpha)$ and $y(\alpha)$ do not necessarily need to be increasing with α . We now prove that all contingent menu mechanisms are truthful and do satisfy many desired properties.

Theorem 12. *Any contingent menu mechanism defined above satisfies (P1), (P2), (P4)-(P6).*

Proof.

- Truthfulness and IR follows from the same argument for that of the second price auctions: utilities are monotonically decreasing as menus increase thus it is a dominant strategy to report the zero-crossing, which is guaranteed to be unique.
- It is easy to see that the mechanism is deterministic and anonymous, and never left the resource unallocated.

□

We should note, however, we are not restricting the domain for the payment menus thus the mechanism is not necessarily BB. This could be easily fixed by restricting the prices to the first quadrant and not including mechanisms like SP+C.

It might not be clear whether a CMM with non-singleton menus exists, however, we show the existence by construction through the following example of a two-payment mechanism.

Definition 10 (Two-choice, contingent-menu mechanism, CMM(q)). This is a DRM. Collect reported type \hat{F}_i from each agent, and compute corresponding utility $\hat{u}_i(z, y)$ for penalty z and base payment y . Define a set of payment menus, $\Pi = \{\pi_\alpha\}_{\alpha \in \mathbb{R}^+}$ where each menu consists of two choices $\pi_\alpha = \{(q\alpha, 0), (0, \alpha)\}$, for parameter $q > 0$. Given a menu π_α , define $\hat{u}_i(\pi_\alpha) \triangleq \max_{(z, y) \in \pi_\alpha} \hat{u}_i(z, y)$. Let the *zero-crossing menu* for agent i , α_i , be the maximum α such that $\hat{u}_i(\pi_\alpha) \geq 0$.³

- Allocate the resource in period zero to the agent with the highest zero-crossing menu.
- Consider allocated agent i^* , and the “second best” agent, $i' = \arg \max_{i \neq i^*} \alpha_i$. The allocated agent's two-part payment is $\arg \max_{(z, y) \in \pi_{\alpha_{i'}}} \hat{u}_i(z, y)$.

A larger class of mechanisms which have the two-payment mechanism defined in Definition 10 as a special case is as follows.

Definition 11 (Two choice, contingent-menu mechanism, CMM(q, γ_1, γ_2)). Consider the CMM with the set of menus Π parameterized by non-negative real numbers $\Pi = \{\pi_\alpha\}_{\alpha \in \mathbb{R}^{\geq 0}}$ where each menu $\pi_\alpha \in \Pi$ contains two contingent payment pairs:

$$\pi_\alpha = \left\{ \left(\alpha, \frac{\gamma_1}{1 - \gamma_1} \alpha \right), \left(q\alpha, \frac{\gamma_2}{1 - \gamma_2} q\alpha \right) \right\}$$

The elements of each menu can be seen as “ γ -CSP prices residing on straight lines through the origin with different slopes. $\gamma = 1$ corresponds to SP and $\gamma = 0$ corresponds to CSP (see Appendix D for more discussions). Define total order \succeq on Π as: $\pi_{\alpha_1} \succeq \pi_{\alpha_2}$ iff $\alpha_1 \geq \alpha_2$ (as real numbers). Note that the CMM defined in Definition 10 has $\gamma_1 = 1$ and $\gamma_2 = 0$.

We would next show that this set of payment sets satisfy the assumptions (B1)-(B3).

³We can show that $\hat{u}_i(\pi_\alpha)$ is a continuous and decreasing function of α , thus for each type we can uniquely define α_i .

Proof.

1. Each menu π_α is a discrete set with two elements, thus is closed (B1).
2. Denote $z_1(\alpha) = \alpha$, $y_1(\alpha) = \frac{\gamma_1}{1-\gamma_1}\alpha$, $z_2(\alpha) = q\alpha$, $y_2(\alpha) = \frac{\gamma_2}{1-\gamma_2}q\alpha$. It's easy to see that for any type u , $u(z_1(\alpha), y_1(\alpha))$ and $u(z_2(\alpha), y_2(\alpha))$ are both continuous and strictly decreasing with α , thus the maximum of the two

$$u(\pi_\alpha) = \max(u(z_1(\alpha), y_1(\alpha)), u(z_2(\alpha), y_2(\alpha)))$$

is also continuous and monotonically decreasing with respect to α . (B3).

3. From (A1)-(A3) we know that $u(\pi_0) > 0$ and $\lim_{\alpha \rightarrow \infty} u(\pi_\alpha) < 0$ for any type u , thus there is a unique zero-crossing. (B2).

□

We show in the following example with detailed explanation and illustrations, and also provide examples to show that CMM could work better than CSP by charging a higher penalty, or worse than CSP for certain economies by 1) charging a lower penalty and 2) not allocating to the CSP winner.

Example 9 (CMM(2) v.s. CSP). We consider the following three economies.

- Economy 1: Agent 1: exponential model with $w_1 = 10$, $\lambda_1 = 0.08$. Agent 2: exponential model with $w_2 = 15$, $\lambda_2 = 0.025$. We can compute the bids under CSP and CMM(2) as $b_{1,\text{CSP}} = 3.78$, $b_{2,\text{CSP}} = 2.22$, $\alpha_1 = 2.72$ and $\alpha_2 = 1.41$, as shown in Figure 19.

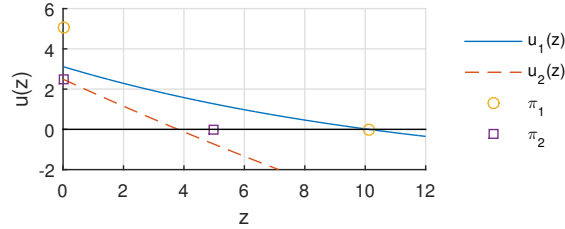


Figure 19: Expected utility curves and CMM payments for agents in economy 1, Example 9 where CMM(2) has better utilization than CSP.

We can see that agent 1 gets allocated the resource under both CSP and CMM(2). Under CSP, agent 1 pays the penalty at the zero-crossing point of agent 2. Under CMM(2), agent 1 faces two possible price in the menu determined by agent 2's report, marked as purple squares in Figure 19. Agent 1 gets a higher utility at the price on the horizontal axis thus is charged this price under CMM(2). The penalty is higher than that under CSP, thus utilization is improved (0.67 under CSP and 0.7 under CMM(2)).

- Economy 2: Agent 1: exponential model with $w_1 = 10$, $\lambda_1 = 0.08$. Agent 2: exponential model with $w_2 = 15$, $\lambda_2 = 0.025$. We can compute the bids as $b_{1,\text{CSP}} = 10.12$, $b_{2,\text{CSP}} = 3.80$, $\alpha_1 = 5.06$ and $\alpha_2 = 2.49$, as shown in Figure 19

We can see that agent 1 gets allocated the resource under both mechanisms again. What is different is that in this economy, between the two prices in the menu π_2 , agent 1 gets a higher utility at the two part payment on the vertical axis, where there is no penalty! The utilization under CMM(2) for this economy is therefore worse than that of CSP, where agent 1 needs to pay

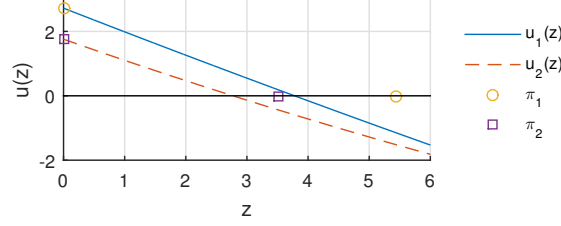


Figure 20: Expected utility curves and CMM payments for agents in economy 2, Example 9 where CMM(2) has worse utilization than CSP.

the zero-crossing of the second agent as the penalty (compare 0.29 under CSP and 0.25 under CMM(2)).

- Economy 3: Agent 1: exponential model with $w_1 = 10$, $\lambda_1 = 0.08$. Agent 2: exponential model with $w_2 = 15$, $\lambda_2 = 0.025$. We can compute the bids under CSP and CMM(2) as $b_{1,\text{CSP}} = 9.44$, $b_{2,\text{CSP}} = 11.27$, $\alpha_1 = 6.80$ and $\alpha_2 = 5.64$, as shown in Figure 19

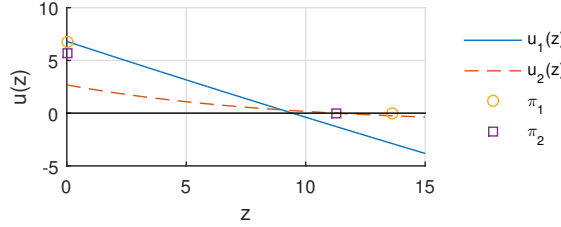


Figure 21: Expected utility curves and CMM payments for agents in economy 3, Example 9 where CMM(2) does not allocate to the CSP winner thus has way worse utilization than CSP.

We can see that in this economy, the resource is allocated to agent 2 under CSP but CMM(2) allocated the resource to agent 1, who has a smaller zero-crossing point but higher zero-crossing α . The utilization is therefore way lower than that under CSP (0.2592 under CMM(2) compared with 0.85 under CSP).

We can see that the reason why CMM is able to achieve a higher utilization than CSP is that it is able to charge the allocated agent in some economies a higher penalty than the second highest zero-crossing. This would not be individually rational for some other allocated agent, as discussed in the proof of Lemma 3. However, by offering some other payments to the allocated agent (and at least one of them must be on the frontier of the rest of the agents in the economy, as we will see), the IR issue is solved since any agent that must get allocated the resource but has negative utility at the high penalty can choose the other payment.

E.3 Simulation Results

We provide in this section simulation results for the contingent menu mechanisms, for various type models and distributions.

E.3.1 Exponential

We first present the simulation results under exponential model.

Recall that under the exponential type model, $\mathbb{E}[V_i] = w_i - 1/\lambda_i$, where w_i is the value for using the resource and $1/\lambda_i$ is the expected opportunity cost. Assume $1/\lambda_i \sim U[0, L]$, parameterized by $L > 0$ and that given λ_i , the value w_i is uniformly distributed $w_i \sim U[0, \lambda_i^{-1}]$. We set $L = 10$, vary the number of agents from 2 to 15, and compute average utilization over 10,000 randomly generated economies. See Figure 22.

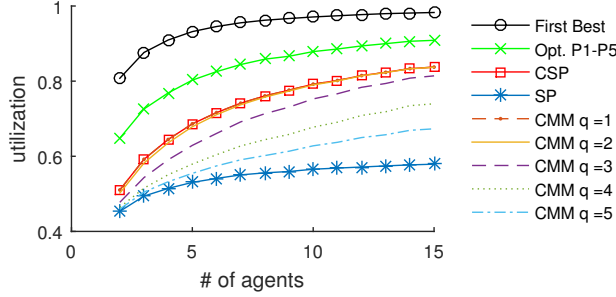


Figure 22: Average utilization under different mechanisms, along with the first-best and “opt under P1-P5” baselines.

The two-choice CMM(q) mechanism is tested with different q parameters, which determines the slope of the segment connecting the choices of two-part payments in each menu and how favorable the $(q\alpha, 0)$ payment is to an agent.

We see that CSP is much better than SP and the gap increases as the number of agents increases. With small q , CMM coincides with CSP since $(q\alpha, 0)$ is preferred by all agents. As q increases, the average utilization decreases, thus the payment menu is not helpful for utilization. The utilization gap between CSP and the best possible utilization under (P1)-(P5) means that, the possibility remains of better mechanisms if we drop (P6) and (P7). We note, however, that the richer CMM mechanisms failed to close the gap. The utilization gap between the first-best (complete information) and the incomplete information baseline shows the cost of self-interest and private information on the ability to achieve a good coordination solution.

Looking in some more depth at CMM(q), we compare the utilization economy-by-economy between CSP and CMM(q) for different q , as in Figure 23. Points above the diagonal line are instances where CMM(q) has higher utilization, whereas points under the diagonal are cases when CSP works better.

All instances are with five agents. We can see that 1) when q is small, utilization under CMM(q) is identical to that of CSP. This is because in any menu of payment $\pi_\alpha = \{(q\alpha, 0), (0, \alpha)\}$, the payment on horizontal axis $(q\alpha, 0)$ is more favorable for any agent type, when $q = 1$:

$$u(\alpha, 0) \geq u(0, \alpha), \forall u, \forall \alpha \geq 0$$

Therefore, the zero-crossing menu, allocation and payments are all defined by the $(q\alpha, 0)$ part of the menus, thus the mechanism behaves exactly as CSP.

As q increases, we see some improvement in utilization when the allocated agent in CSP already has high utilization (thus has shallow utility curves) and prefers the $(q\alpha, 0)$ payment in CMM (which might be higher than the CSP penalty). Lower utilization CSP winners increasingly prefer the other payment $(0, \alpha)$, thus reducing the utilization because there is no penalty. This effect can be observed from Example 9 as well: the allocated agent prefers the $(q\alpha, 0)$ payment only if her probability of paying the penalty is relatively small, which means, her utilization is already high.

We will also observe that as q gets larger, there will be more instances when CMM allocates

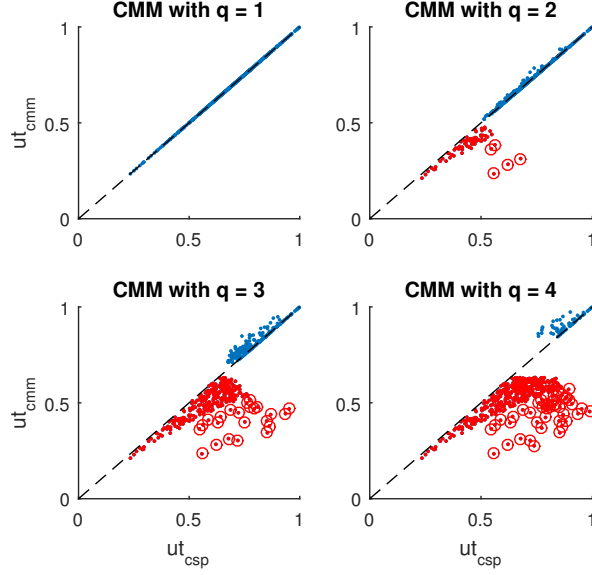


Figure 23: Utilization in CSP vs CMM(q) with different q for the exponential model. Blue dots: CMM is at least as high as CSP. Red dots: CMM is lower than CSP. Red circle: CMM allocates the resource to a different agent from the CSP winner.

the resource to a different agent than the CSP winner (red circles). This would result in utilization a lot lower than that of CSP, which is the case for economy 3 in Example 9.

E.3.2 Uniform Model

We now look at the uniform model as described in Appendix B.2, where each agent's value is assumed to be uniformly distributed on some interval parameterized by $0 < a_2 < a_1$.

$$V_i \sim U[-a_{1,i}, a_{2,i}]$$

Assume that $a_{1,i}$'s are iid uniformly distributed on $[0, 10]$ and that given $a_{1,i}$, $a_{2,i}$ is uniformly distributed on $[0, a_{1,i}]$ (s.t. $\mathbb{E}[V_i] < 0$ and dominant strategies exist).

Average utilization over 10,000 iterations of various mechanisms and benchmarks are presented in Figure 24. Similar to the results that we see for exponential distribution, there is a gap between the upper bound of any mechanism satisfying (P1)-(P5), however, the simple family of CMM(q) mechanism is unable to close the gap.

E.3.3 The (w_i, p_i) Model

We now present the average utilization of different mechanisms when agents' types are of the simple (w_i, p_i) model as introduced in Example 1. We use the most simple distribution, where w_i 's i.i.d. uniformly distributed on $[0, 10]$ and p_i 's are i.i.d. uniformly distributed on $[0, 1]$. Average utilization over 10,000 iterations as plotted in Figure 25.

Observe that the utilization of CSP and the optimal benchmark under P1-P5 coincides. This is because the utilization of a (w_i, p_i) agent cannot be improved by penalties, thus the best any mechanism satisfying (P1)-(P5) is able to do is to select the agent at the zero-crossing of the frontier and reserve prices and CMM's would not be helpful.

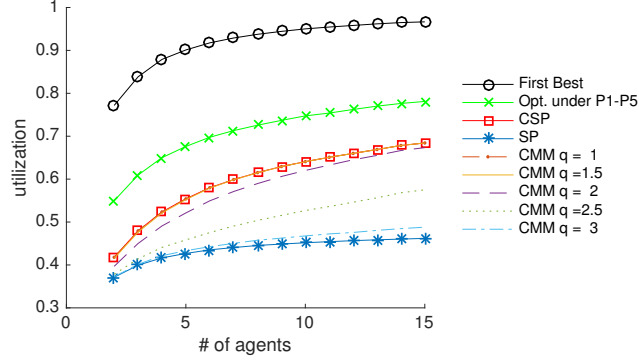


Figure 24: Utilization of CMM(q) mechanism, compared with SP, CSP and benchmarks for the uniform model

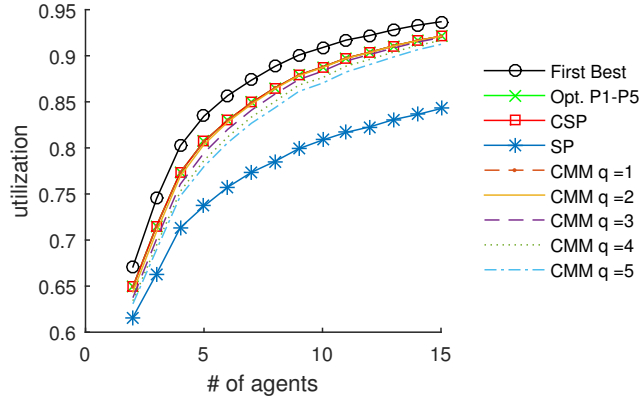


Figure 25: Utilization of CMM(q) mechanism, compared with SP, CSP and benchmarks for (w_i, p_i) Model